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Toward a Network Coding Constellation for Two-Way Relay Node Channels

Eugène David Ngangue Ndih, and Soumaya Cherkaoui
Department of Electrical and Computer Engineering
Université de Sherbrooke
Québec, Canada
Email: {eugene.ngangue, soumaya.cherkaoui}@usherbrooke.ca

***Abstract*—In Physical-Layer Network Coding (PNC), the symbol detected at the relay node in two-way relay channels (TWRC) is the superposition of the symbols transmitted by the sinks. We refer to these symbols as pnc-symbols. In traditional modulation constellations such as Quadrature Amplitude Modulation (QAM) and Phase Shift Keying (PSK), the demodulator may fail to optimally identify pnc-symbols formed from pairs of central symmetrical symbols (CSS) because their superposition may yield to the same output point (i.e., to the same decision region) due to the presence of a central symmetry point. In order to avoid such ambiguity, we propose a non Central Symmetry Constellation (CSC), called 4-TRAQAM, which is used by the sinks such that the decision regions of the output points are pair-wise disjoint. We show that the sinks in the PNC-based TWRC can be considered as a single transmitter using the same modulation with a higher order (10-TRAQAM). We further derive the average energy of pnc-symbols and the error probability of the derived PNC-based TWRC modulation, and we show that the 4-TRAQAM provides the best trade-off between the average energy and the disjointness of the decision regions, compared to other used QAM.**

I. Introduction

Recently, cooperative communication has gained much interest in the research community to optimize network throughput in wireless networks. In general, cooperative communication techniques can be grouped into three different categories depending on how the users share their resources: frequency sharing, antenna sharing and relay node sharing.

Traditionally in relay node based cooperative communications, the relay node receives packets from sinks at different time slots to avoid interference and then forwards the different packets received separately at different other time slots. With the concept of network coding (NC), Ahlswede *et al.* [1] improved the functionalities of the relay node. Indeed, rather than simply copy and forward the signal received from a sink to the destination node, in NC, the relay node can re-encode (XOR) the signals received from multiple sinks and forwards the resulting coded signal to the destination nodes. By this way, the bandwidth usage is improved compared to the traditional relaying scheme. Further improvements in the bandwidth usage can be observed both with PNC [2] and analog network coding (ANC) [3], where the coded information forwarded by the relay node is obtained through the interference caused by the simultaneous transmissions of the sinks. However, such improvement is obtained at the expense of decoding complexity at the destination node.

Basically, to avoid forwarding corrupted packets due to incorrect decoding, the intermediate node in such interference-based NC systems does not try to decode the content of individual messages. In [3], the authors used the amplify-and-forward (AF) [4] relaying technique. The relay node simply amplifies the interference signal and forwards it to the destination node which extracts the desired packet by mean of self interference. In [2], S. Zhang *et al.* used the decode-and-forward (DF) [4] as relaying strategy at the relay node. In this case, the relay node partially decodes the interference signal and retransmits it using an appropriate symbol alphabet in the case of Binary Phase Shift Keying (BPSK). The destination node can therefore recover the packet of interest by XORing the received packet with its own packet as it is the case in NC. The authors in [5] extended the work in [2] to the case of Quadrature Phase Shift Keying (QPSK). In [6], Koike-Akino *et al.* used denoise-and-forward (DNF) as relaying technique. In this cooperative technique, the relay node removes the noise in the interference signal such that it is not amplified during the signal amplification prior to the retransmission. As in AF, the destination node creates a self interference to extract the desired signal. Although these techniques gain a lot of interest in cooperative communications, they may be suboptimal for all-to-all communications, such as content distribution in peer-to-peer (P2P) networks, or information dissemination among vehicles in an area since each node, including the intermediate node, is interested in the content of each individual message.

In this work, we address the issue of full decoding interfering symbols at the relay node in PNC-based TWRC for all-to-all communications. More specifically, we address the problem of the choice of the modulation constellation lattice at the transmitter nodes which guarantees disjoint decision regions of the pnc-symbols at the receiver constellation in order to exploit the decision rule based on decision regions (DRDR) of the optimum demodulator. To the best of our knowledge, this is the first time this approach is reported in the literature in the case of PNC-based TWRC. For this purpose, we consider the same assumptions as in [2] for simplicity of illustration, that is, a perfect synchronization between sinks transmissions in reciprocal non distortive additive white Gaussian noise (AWGN) channels. In PNC-based TWRC, the receiver constellation is obtained from the pair-wise vectorial summation of the input constellations. As a consequence, due to the presence of a central symmetry point in traditional modulation, such as QAM or PSK, the DRDR-based demodulator may fail to optimally identify pnc-symbols formed from pairs of CSS because their superposition may yield to the same output point, thus to the same decision region.

Our contribution is threefold:

- To avoid the superposition of decision regions at the receiver constellation in PNC-based TWRC, we propose a non CSC at each sink obtained by adequately changing the symbol values of the conventional 4QAM, such that the decision regions of all the possible output points at the relay node constellation are pair-wise disjoint. The advantages of using such an approach have been reported in previous works [7] noting that the symmetry of a constellation could be broken by changing the values and/or the probability of the symbols.
- We derive the average energy of pnc-symbols at the receiver constellation and we show that trapezoid constellations provide the best trade-off between the average energy of pnc-symbols and the disjointness of the decision regions, compared to rectangular QAM (RQAM) and square QAM (SQAM) which provide the minimum average energy of pnc-symbols at the expense of superposed decision regions.
- We show that the transmitters in the PNC-based TWRC can be considered as a single transmitter using the same modulation with a higher order and we derive the probability of error of the derived modulation.

The remainder of the paper is organized as follows. In Section II, we present the system model and some assumptions used throughout the paper. Section III presents an optimized constellation for pnc-symbols detection and decoding in the case of 4-ary. The performance evaluation in terms of average energy of pnc-symbols and probability of error of the derived pnc-constellation is presented in Section IV. Some concluding remarks and future work are presented in Section V.

II. System model and assumptions

Lets consider the three-node network, also known as TWRC, illustrated in Fig. 1. It consists of two sinks (S_1)

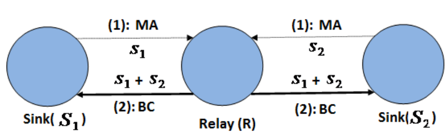

Fig. 1. The TWRC: (1) is the first stage and (2) is the second stage.

and (S_2) who want to exchange information via a relay node (R). In this work, we consider a scenario where the relay node is also interested in the content of the sinks' individual messages. Such scenarios may be encountered in all-to-all communications where each node is interested in the messages of the other nodes. For example, during routing discovery and routing update phases in wireless ad hoc networks, or in the case of alert message dissemination to all vehicles in an alert zone in a vehicular ad hoc network (VANET).

Each node is supposed to be equipped with an omni-directional antenna, and the communication between two nodes is half duplex so that transmission and reception at any node cannot occur in the same time slot.

PNC in TWRC is divided into two main stages: the first stage or multiple access (MA) stage and the second stage or broadcast (BC) stage. In the former stage, the sinks send their packets to the relay node. Basically, depending on the application interest, the relay node can choose in the BC stage either to decode-and-forward (DF) [2] or to amplify-and-forward (AF) [3] the received signal.

In this work, we focus on full decoding interfering symbols at the relay node. For simplicity of analysis, we consider reciprocal AWGN channels with no distortion, and we suppose perfect synchronization of network nodes. We further assume that the signals have the same power level.

III. The 4-TRAQAM Constellation for PNC in TWRC

Researchers have always looked for the optimum modulation constellation that may provide better performance in terms of probability of error, demodulation complexity, bandwidth usage, energy consumption, etc. For instance, Foschini *et al.* [8] look for minimum probability of error modulation in Gaussian noise. This work aims to investigate a modulation constellation which will be appropriate for PNC in TWRC. For simplicity of illustration and without loss of generalization, we restrict this study to the case of QAM modulation with $k = \log_2(M) = 2$ bits per symbol. In other words, we consider as input symbol alphabet at a transmitter, the set $\mathcal{I}$ given by

$$\mathcal{I} = \Big\{\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4\Big\}, \tag{1}$$

where $\mathbf{s}_m = \big(s_{m,I}, s_{m,Q}\big)$ for $m = 1, 2, 3, 4$, is the transmitted symbol vector in the in phase and quadrature (I, Q) representation. Lets consider the MA phase in the TWRC depicted in Fig. 1 with the system model described in Section II. Throughout the paper, the subscripts I and Q refer to the in-phase component and the quadrature component respectively. For $i = 1, 2$, let $s^{(i)}(t)$ the signal transmitted by the sink (S_i) be given by

$$\begin{aligned} s^{(i)}(t) &= \mathrm{Re}\Big[\Big(s_I^{(i)} + js_Q^{(i)}\Big)g(t)e^{j2\pi f_c t}\Big], \quad 0 \le t \le T \\ &= s_I^{(i)} g(t)\mathrm{cos}2\pi f_c t - s_Q^{(i)} g(t)\mathrm{sin}2\pi f_c t \end{aligned} \tag{2}$$

where $s_I^{(i)}$ and $s_Q^{(i)}$ are the information-bearing signal amplitudes of the quadrature carriers of the node (S_i), f_c is the carrier frequency, T is the symbol interval and g(t) is the pulse shape signal. Assuming AWGN channel with no distortion and perfect synchronization of the transmitters, the signal received at the relay node can be expressed as

$$\begin{aligned} r(t) &= s^{(1)}(t) + s^{(2)}(t) + n(t) \\ &= \Big(s_I^{(1)} + s_I^{(2)}\Big)g(t)\mathrm{cos}2\pi f_c t \\ &\quad -\Big(s_Q^{(1)} + s_Q^{(2)}\Big)g(t)\mathrm{sin}2\pi f_c t + n(t) \end{aligned} \tag{3}$$

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where $n(t)$ is a zero-mean stationary white Gaussian process with variance N_0. Let us denote $s_I = s_I^{(1)} + s_I^{(2)}$ and $s_Q = s_Q^{(1)} + s_Q^{(2)}$. Therefore, the signal received at the relay node can be expressed as

$$r(t) = s_I g(t)\cos 2\pi f_c t - s_Q g(t)\sin 2\pi f_c t + n(t) \quad (4a)$$
$$= s(t) + n(t), \quad (4b)$$

where

$$s(t) = s_I g(t)\cos 2\pi f_c t - s_Q g(t)\sin 2\pi f_c t. \quad (5)$$

Equation (5) corresponds to the form of a signal $s(t)$ transmitted using a two quadrature carriers modulation such as QAM. In (2), the information-bearing signal amplitudes $s_I^{(i)}$ and $s_Q^{(i)}$ are defined such that the vector signal $\mathbf{s}^{(i)} = \left(s_I^{(i)}, s_Q^{(i)}\right)$ is an element of the 4QAM symbol alphabet $\mathcal{I}$ defined in (1). Therefore, the transmitted vector signal $\boldsymbol{s}$ can be expressed as

$$\boldsymbol{s} = \mathbf{s}^{(1)} + \mathbf{s}^{(2)}. \quad (6)$$

As a result, we have the following proposition:

Proposition 1. *The transmission, over an AWGN TWRC with no distortion, from two nodes (S_1) and (S_2) perfectly synchronized with equal transmit power and using the same $M-ary$ modulation to a relay node (R) is equivalent to the transmission of a single node (S) to the relay node (R) with the usage of an equivalent $M_\Sigma - ary$ modulation, where*

$$M_\Sigma = \frac{M(M+1)}{2}. \quad (7)$$

Proof: From (5) and (6), it follows that the equivalent node (S) uses the same modulation technique as (S_1) and (S_2). Moreover, from (6), it follows that the number of possible different symbol interferences, referred to as pnc-symbols, that can be formed at the relay node is given by $\binom{M}{1} + \binom{M}{2} = \frac{M(M+1)}{2}$. ■

For instance, in the case where 4QAM is used as modulation scheme by the sinks (S_1) and (S_2), the equivalent node (S) will use a 10QAM pnc-symbol alphabet $\mathcal{I}_{PNC}$ given by

$$\mathcal{I}_{PNC} = \left\{\boldsymbol{s}_{11}, \boldsymbol{s}_{12}, \boldsymbol{s}_{13}, \boldsymbol{s}_{14}, \boldsymbol{s}_{22}, \boldsymbol{s}_{23}, \boldsymbol{s}_{24}, \boldsymbol{s}_{33}, \boldsymbol{s}_{34}, \boldsymbol{s}_{44}\right\} \quad (8)$$

where

$$\boldsymbol{s}_{ij} = \boldsymbol{s}_{ji} = \mathbf{s}_i^{(1)} + \mathbf{s}_j^{(2)} = \mathbf{s}_j^{(1)} + \mathbf{s}_i^{(2)}. \quad (9)$$

From (9) one notices that the constellation lattice of the pnc-symbol alphabet set $\mathcal{I}_{PNC}$ used by the equivalent node for transmission is built upon the pair-wise summation of symbol vectors in the symbol set alphabet $\mathcal{I}$ given in (1) in the case for instance of 4QAM modulation. As a consequence, judicious selection of the lattice constellation is required at the sinks (S_i) such that the resulting lattice constellation of the equivalent node referred to as the *pnc-constellation* will provide better performance in terms of probability of error, detection and decoding complexity of the pnc-symbols, bandwidth usage, and so. Hence, we have the following proposition:

Proposition 2. *The detection and decoding of pnc-symbols require either appropriate decision rule other than the DRDR in cases of CSC lattices such as SQAM and RQAM lattices, or an appropriate non CSC lattice at the sinks such that the DRDR can be used without change at the relay node.*

Proof: In the conventional DRDR, the decision regions must be pair-wise disjoint for the optimum demodulator. From (9), one may notice that in M-ary SQAM and RQAM, there exist at least two-tuples of CSS vectors $\left(\mathbf{s}_i^{(1)}, \mathbf{s}_j^{(2)}\right)$ and $\left(\mathbf{s}_k^{(1)}, \mathbf{s}_l^{(2)}\right)$ with $i \neq j \neq k \neq l$, such that the resulting pnc-symbol vectors $\boldsymbol{s}_{ij}$ and $\boldsymbol{s}_{kl}$ are equal and have the same decision region. Therefore, to generate an equivalent constellation with non completely/partially overlapping decision regions, the constellation at the sinks must be non central symmetrical. ■

In this work, we are interested in the latter case. We propose a non CSC 4-ary trapezoid constellation lattice, which we refer to as the 4-TRAQAM. In this 4-TRAQAM, the decision regions of the pnc-symbols in the derived 10-TRAQAM pnc-constellation are pair-wise disjoint as shown in Fig. 2.

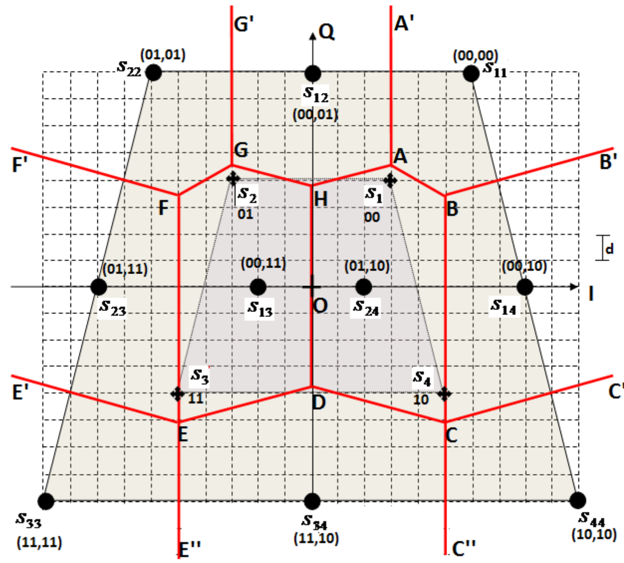

Fig. 2. Decision regions of the 10-TRAQAM pnc-constellation generated by the superposition of two 4-TRAQAM constellations.

IV. Performance Analysis of the 10-TRAQAM pnc-constellation

A. Average Energy $\mathcal{E}_\Sigma$ of pnc-symbols

Assuming that the bits are identically distributed, the average energy $\mathcal{E}_\Sigma$ of pnc-symbols in the 10-TRAQAM pnc-constellation is the average of the squared distance D_{ij} of the constellation points $\mathbf{s}_{ij} = \left(s_{ij,I}, s_{ij,Q}\right)$ from the origin $O(0,0)$ of the (I, Q) plane. That is,

$$D_{ij} = \mathrm{d}(O, \mathbf{s}_{ij}) \quad (10)$$

where $\mathrm{d}(A, B)$ is the Euclidean distance between the points A and B. As appearing in Fig. 2, the minimum distance D_0 corresponds to

$$D_0 = \mathrm{d}(\mathbf{s}_{13}, \mathbf{s}_{24}) = 4d \quad (11)$$

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where d is a distance unit as shown in Fig. 2. We have

$$\mathcal{E}_\Sigma = \frac{1}{M_\Sigma}\sum_{\boldsymbol{s}_{ij}\in\mathcal{I}_{PNC}} D_{ij}^2 = \frac{1}{M_\Sigma}\sum_{i=1}^{M}\sum_{j=i}^{M} D_{ij}^2 \quad (12)$$

Therefore, the average energy $\mathcal{E}_\Sigma$ of pnc-symbols in the 10-TRAQAM pnc-constellation is

$$\mathcal{E}_\Sigma = \frac{99}{20}D_0^2. \quad (13)$$

In addition, the average energy $\mathcal{E}_s$ of source symbols in the originating 4-TRAQAM constellation is given by

$$\mathcal{E}_s = \frac{1}{M}\sum_{k=1}^{M} d_k^2 \quad (14)$$

where $d_k = \mathrm{d}(O, \mathbf{s}_k)$, and $\mathbf{s}_k \in \mathcal{I}$. Given that

$$D_{ij}^2 = d_i^2 + d_j^2 + 2d_i d_j \cos(\Phi_{ij}) \quad (15)$$

where Φ_{ij} is the angle between the symbol vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ of $\mathcal{I}$. Substituting (15) into (12), we obtain

$$\mathcal{E}_\Sigma = 2\Big(1 + 2\frac{M}{M_\Sigma}\Big)\mathcal{E}_s + \frac{2}{M_\Sigma}\sum_{i=2}^{M}\sum_{j=1}^{i-1} d_i d_j \cos(\Phi_{ij}). \quad (16)$$

The expression in (16) is common to all originating quadrilateral 4QAM constellation lattices. Only the second term, denoted as ΔE_s, significantly affects the average energy of the pnc-symbols. To see how the shape of the primary 4QAM constellation lattice affects the average energy of the pnc-symbols, let us make the following simple analysis. Consider Fig. 3

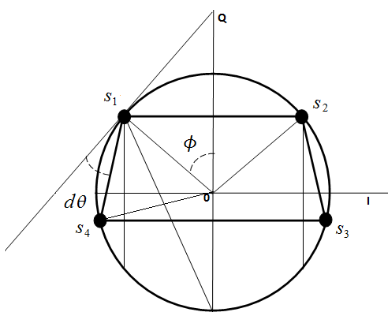

Fig. 3. Shapes of axial symmetry quadrilateral 4QAM constellation. The symbols s_1, s_2, s_3 and s_4 can move on the circle to vary the shape of the constellation lattice.

where all distances d_i are equal to a constant R. The angles Φ_{ij} are such that $\Phi_{12} = 2\phi$, $\Phi_{13} = 2\phi + 2d\theta$, $\Phi_{14} = 2d\theta$, $\Phi_{23} = 2d\theta$, $\Phi_{24} = 2\phi + 2d\theta$ and $\Phi_{34} = 2\pi - 2\phi + 4d\theta$. Therefore, a simple analysis of the second term ΔE_s shows that for a given ϕ, the minimum of the average energy of pnc-symbols $\mathcal{E}_\Sigma$ is obtained in the cases of square and rectangle lattices as shown in Fig. 4. However, with a good choice of the angles between symbols, a good trade-off between pnc-symbol detections and average energy of pnc-symbols can be met with trapezoids, since the latter provide much flexibility.

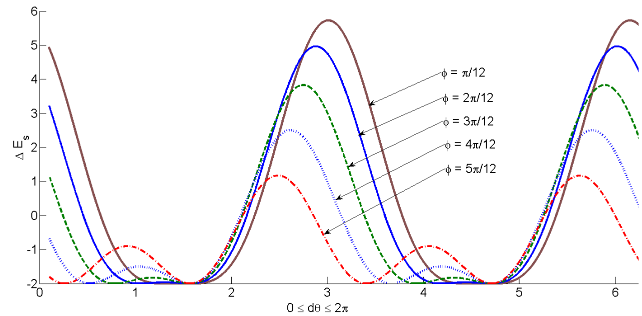

Fig. 4. Variation of ΔE_s with respect to $d\theta$ for different ϕ.

B. The error probability

Assuming that the pnc-symbols are equally likely, the symbol error probability (SEP), $P(e)$, can be written as [9]

$$P(e) = 1 - \frac{1}{M_\Sigma}\sum_{\boldsymbol{s}_{ij}\in\mathcal{I}_{PNC}} P(\mathbf{r}\in\mathcal{R}_{ij}|\boldsymbol{s}_{ij}) \quad (17)$$

where M_Σ is given in (7). $P(\mathbf{r}\in\mathcal{R}_{ij}|\mathbf{s}_{ij})$ is the probability that the received symbol vector $\mathbf{r}$, which is the interference of the symbols $\mathbf{s}_i^{(1)}$ and $\mathbf{s}_j^{(2)}$ transmitted by the sinks (S_1) and (S_2) respectively, is in the decision region $\mathcal{R}_{ij}$ for the pnc-symbol $\boldsymbol{s}_{ij}$. $\mathcal{R}_{ij}$ is represented in Fig. 2 by the delimited area which contains the corresponding pnc-symbol $\boldsymbol{s}_{ij}$. For example, the decision region of the pnc-symbol $\boldsymbol{s}_{11}$ is the area delimited by $A'ABB'$, the decision region of the pnc-symbol $\boldsymbol{s}_{24}$ is the area delimited by $ABCDHA$, and so on.

Let $P_{\mathbf{s}_{ij}} \triangleq P(\mathbf{r}\notin\mathcal{R}_{ij}|\mathbf{s}_{ij})$ denote the probability of symbol error when the equivalent transmitted signal vector is $\mathbf{s}_{ij}$. The probability $P_{\mathbf{s}_{ij}}$ can be expressed as [10]

$$P_{\mathbf{s}_{ij}} = \frac{1}{2\pi}\sum_{k=1}^{N_{ij}}\int_0^{\theta_{k,ij}} \exp\Big[-\frac{x_{k,ij}^2\sin^2(\Psi_{k,ij})}{N_0\sin^2(\theta+\Psi_{k,ij})}\Big]d\theta \quad (18)$$

where N_{ij} is the number of vertices of the polygon representing the decision region $\mathcal{R}_{ij}$. N_{ij} also corresponds to the number of sided decision regions of the decision region $\mathcal{R}_{ij}$, that is the angular sector delimited by the segment line linking two adjacent vertices and two semi-infinite lines passing through the given vertices and the point representing the symbol vector. The angle $\theta_{k,ij}$ represents the angle of the angular sector k. The distance $x_{k,ij}$ is the distance between the symbol representing the transmitted signal vector s_{ij} and the k^{th} vertex in the decision area $\mathcal{R}_{ij}$. This is illustrated in Fig. 5 describing the partitioning of an example of decision region $EFIP$ of a symbol $\mathbf{s}_{11}$. Additional information and more details about this approach can be found in [10].

Given that $P(\mathbf{r}\in\mathcal{R}_{ij}|\mathbf{s}_{ij}) = 1 - P_{\mathbf{s}_{ij}}$, and substituting (18) into (17), one obtains

$$P(e) = 1 - \frac{1}{M_\Sigma}\sum_{\boldsymbol{s}_{ij}\in\mathcal{I}_{PNC}}\Bigg(1 - \frac{1}{2\pi} \times \sum_{k=1}^{N_{ij}}\int_0^{\theta_{k,ij}} \exp\Big[-\frac{x_{k,ij}^2\sin^2(\Psi_{k,ij})}{N_0\sin^2(\theta+\Psi_{k,ij})}\Big]d\theta\Bigg) \quad (19)$$

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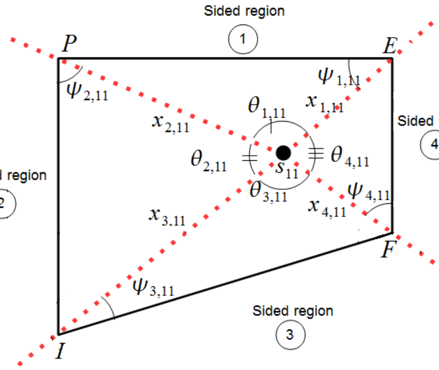

Fig. 5. Example of geometrical description of a decision region.

That is,

$$P(e) = \frac{1}{M_\Sigma}\frac{1}{2\pi}\sum_{\boldsymbol{s}_{ij}\in\mathcal{I}_{PNC}}\sum_{k=1}^{N_{ij}}\Big\{ \int_0^{\theta_{k,ij}} \exp\Big[-\frac{x_{k,ij}^2 \sin^2(\Psi_{k,ij})}{N_0 \sin^2(\theta+\Psi_{k,ij})}\Big] d\theta \Big\} \quad (20)$$

In (20), $x_{k,ij}^2$ can be related to the average energy of pnc-symbols $\mathcal{E}_\Sigma$. Indeed, if $x_{k,ij} = \mathrm{d}(\mathbf{a}_k, \mathbf{s}_{ij})$, where $\mathbf{a}_k(a_{k,I}, a_{k,Q})$ is the vector representing the kth vertex in the decision region $\mathcal{R}_{ij}$, we have

$$\begin{aligned} x_{k,ij}^2 &= (a_{k,I} - s_{ij,I})^2 + (a_{k,Q} - s_{ij,Q})^2 \\ &= \Big[(\hat{a}_{k,I} - \hat{s}_{ij,I})^2 + (\hat{a}_{k,Q} - \hat{s}_{ij,Q})^2\Big]\frac{D_0^2}{16} \\ &= \hat{x}_{k,ij}^2 \frac{D_0^2}{16} = \frac{5}{396}\hat{x}_{k,ij}^2 \mathcal{E}_\Sigma \quad (21) \end{aligned}$$

where $\hat{x}_{k,ij}^2 = (\hat{a}_{k,I} - \hat{s}_{ij,I})^2 + (\hat{a}_{k,Q} - \hat{s}_{ij,Q})^2$ and $\forall m, \exists! \hat{m}$, such that $m = \hat{m}d = \hat{m}\frac{D_0}{4}$. Substituting (21) into (20), we obtain

$$P(e) = \frac{1}{M_\Sigma}\frac{1}{2\pi}\sum_{\boldsymbol{s}_{ij}\in\mathcal{I}_{PNC}}\sum_{k=1}^{N_{ij}}\Big\{ \int_0^{\theta_{k,ij}} \exp\Big[-C\gamma_\Sigma \frac{\hat{x}_{k,ij}^2 \sin^2(\Psi_{k,ij})}{\sin^2(\theta+\Psi_{k,ij})}\Big] d\theta \Big\}, \quad (22)$$

where $C = \frac{5}{396}$ and $\gamma_\Sigma = \frac{\mathcal{E}_\Sigma}{N_0}$ is the pnc-symbol-to-noise ratio.

Fig. 6 shows the comparison of the theoretical SER with the simulated SER of the 10-TRAQAM. One may notice the perfect matching of the curves, emphasizing the correctness of the derived pnc-symbol error probability. In addition, Fig. 6 shows how performant is the 10-TRAQAM modulation with respect to the pnc-symbol-to-noise ratio.

V. Conclusion and Future Work

In this work, we have presented TRAQAM modulation; a non CSC modulation technique which addresses the problem of the superposition of decision regions in pnc-constellations due to the use of CSC modulations (SQAM or RQAM). The TRAQAM modulation makes feasible the use of DRDR for

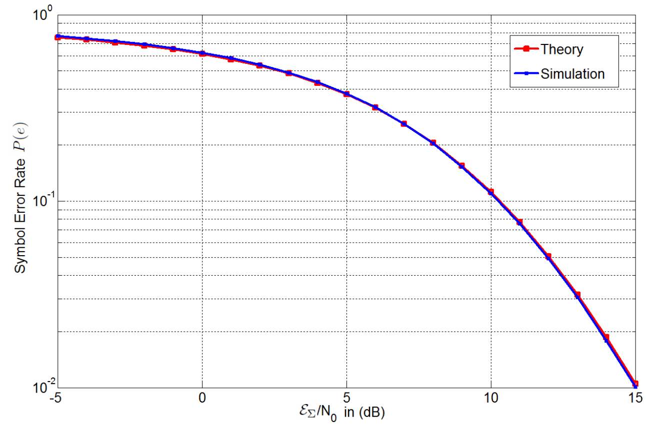

Fig. 6. Comparison of the theoretical SER with the simulated SER of the 10-TRAQAM. The variation of the SER is obtained with respect to the pnc-symbol-to-noise ratio variation.

optimal detection of pnc-symbols in TWRC. The performance analysis of the average energy of pnc-symbols shows that 4-TRAQAM provides the best trade-off between the average energy of pnc-symbols and the disjointness of the decision regions, compared to RQAM and SQAM which provide the minimum average energy of pnc-symbols at the expense of superposed decision regions. We have also shown that the sinks in PNC-based TWRC can be considered as a single transmitter using the same modulation, with a higher order (the 10-TRAQAM).

This new approach of PNC deserves much attention since it can be used together with an appropriate channel coding to fully decode and separate interfering signals at the relay node. This defines the future research directions of the authors with an extension to non reciprocal distortive channels.

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