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Statistical Modeling of the Physical-Layer Network Coding in Time-Varying Two-Way Relay Channels

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***Abstract*—Recent studies have shown that network coding (NC) directly applied at the physical layer – the physical layer network coding (PLNC) – is a promising technique for two-way relay channels (TWRC). Unfortunately, to date, most existing PLNC works have mainly addressed cases of time-invariant channels rather than more realistic multipath time-varying wireless channels. To make further progress in understanding how to optimally implement and design PLNC techniques, we present in this paper, a simple but efficient equivalent interference channel (EIC) model for time-varying TWRC with the flat fading condition. The model captures important parameters such as the phase difference due to the asymmetry of the TWRC in real environments. Based on the EIC model, we derive and analyze some important analytical expressions for the statistical properties of the processes that describe the fading at the different stages of the PLNC in TWRC.**

I. Introduction

Given the broadcast nature of radio transmission in wireless communication, signals from multiple sources often arrive simultaneously at the receiver, causing interference. This interference causes the desired signal to be scrambled with the other signals, thus resulting to a nuisance that most communication designs attempt to either avoid or mitigate. Instead of treating interference as a negative effect to be eliminated, recent techniques inspired from the concept of NC [1] embrace interference to improve communication performance in cooperative networks. In fact, contrary to traditional cooperative schemes, where the relay nodes forward at different time slots the signals received separately from multiples sinks, in NC, the relay nodes mix these different received signals, and forward the resulting signal, thus reducing the bandwidth usage and improving the network throughput, at the price of increasing the decoding complexity at the receiver. Further improvements are observed in PLNC where the mixed signal is obtained through interference. In other words, the number of time slots required to obtain the desired mixed signal at the relay node is reduced in PLNC, since the multiple sinks are encouraged to transmit in the same time slot. A major issue with this interference-based technique is how to optimally design to be effective in realistic time-varying environments. Until now, the main PLNC techniques presented in the literature [2], [3] are based on assumptions which restrict their applications. In the seminal work [2], the authors consider a TWRC and present a PLNC scheme based on symbol-level, carrier-phase, carrier-frequency and time synchronization between the two transmitters. The authors further assume coherence detection at the receiver. In [3], Katti *et al.* present a PLNC technique referred to as analog network coding (ANC) in which the lack of synchronization between the interfering signals is leveraged by means of an interference-free bits-based algorithm which consist in the use of a randomization scheme similar to 802.11 MAC. In other words, ANC encourages delayed transmissions in the same time slot to facilitate the decoding through self-interference, rendering the technique unsuitable in high data rate applications or time-varying channels. Based on these observations, it appears necessary to provide new proposals for modeling interference in more realistic multipath time-varying environments for practical PLNC techniques.

In this paper, we present a simple and efficient equivalent interference channel (EIC) model of TWRC with a flat fading assumption. The channel captures parameters such as the phase and the amplitude differences of the transmitted signals. Based on this model, we derive and analyze the probability density function (PDF), the mean value and the variance of the processes describing the fading at the PLNC stages. By means of extensive simulations, we further show the validity of the model and the accuracy of the analytical expressions of the statistics derived. These results thus provide a statistical insight for PLNC stages to make further progress in understanding how to optimally design and implement such interference-based techniques in realistic environments.

The remainder of the paper is organized as follows. In Section II, we present the system model and some assumptions used throughout the paper. Section III and Section IV respectively present the analytical expressions of the fading processes statistics in the first stage and in the second stage. In Section V, the simulation results are presented and analyzed. The work is finally concluded in Section VI.

II. The System Model

Also known as the three-node wireless network, the TWRC, depicted in Fig. 1, is a basic unit for cooperative transmission in decentralized wireless networks. It has been extensively studied in the literature when evaluating the performance of PLNC techniques [2]–[4]. The TWRC consists of two sinks (S_1) and (S_2) who want to exchange information via a relay node (R). Each node is equipped with an omni-directional antenna, and the communication between two nodes is half duplex so that transmission and reception at any node cannot occur in the same time slot.

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PLNC in TWRC has two main stages. In the first stage or time slot, the sinks send their packets to the relay node. Depending on the Signal-to-Noise ratio (SNR), the relay node can choose to decode-and-forward (DF) [2] or to amplify-and-forward (AF) [3] the received signal in the second stage.

In this work, we consider the AF cooperative strategy although the statistics derived for the first stage can be used for both the DF and AF relaying techniques. We assume a low mobility communication scenario where the sinks move in such a way that there is no difference between the Doppler frequencies of the line-of-sight (LOS) components in both sides of the relay node. We further assume that both the environment (scatters, reflectors, etc.) and the relay node are stationary; the time-varying nature of the channel is due only to the mobility of the sink.

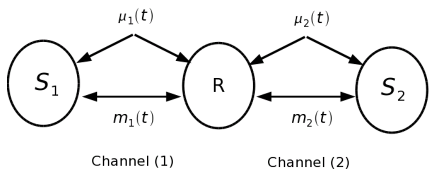

Fig. 1. Asymmetric time-varying two-way relay channel. When it exists, $m_i(t)$ is the LOS component, and $\mu_i(t)$ is the scattered or non LOS (NLOS) component that describes the fading in channel (i).

III. Statistical Modeling of the First Stage

Let $y(t)$, in the complex baseband, be the total signal received at the relay node in the first stage. For analysis purpose, we assume that the relay node can receive signals from the sinks regardless of the difference in the amplitude of the transmitted signals $|x_i(t)| = \frac{P_i(t)}{d_i^{\lambda}(t)}$, where $P_i(t)$, $d_i(t)$ and λ are the transmit power, the distance to the relay, and the path-loss exponent of channel (i) of the transmitter (S_i), respectively. Due to simultaneous transmissions through channel (1) and channel (2), $y(t)$ can be expressed as $y(t) = y_1(t)+y_2(t)$, where $y_i(t)$, for $i = 1, 2.$, represents the signal received at the relay node due only to the transmission through the channel (i). Also in the complex baseband, let $x_i(t)$ be the signal transmitted through the channel (i). Without loss of generality, we assume that $x_2(t)$ can be expressed in term of $x_1(t)$ as $x_2(t) = \alpha x_1(t)$, where α is a complex number whose magnitude $|\alpha|$ and phase θ, represent respectively the ratio of transmitted signals amplitudes, i.e., $|\alpha| := \frac{|x_2(t)|}{|x_1(t)|}$, and the phase difference between the transmitted signals. Therefore, α can be expressed as

$$\alpha := |\alpha|\, e^{j\theta} \tag{1}$$

Let $m_i(t) = \rho_i e^{j\left(2\pi f_{\rho_i} t + \theta_{\rho_i}\right)}$ be the LOS component in channel (i). We assume that $m_i(t)$ is deterministic such that the amplitude or modulus ρ_i, the Doppler frequency f_{ρ_i} caused by the motion of the sink (S_i), and the phase θ_{ρ_i} are fixed values within the duration of the stage. And let $\mu_i(t)$ be the scattered component or process describing the fading in channel (i). $\mu_i(t)$ is modeled as a zero-mean complex Gaussian process with variance $2\sigma_i^2$, i.e., $\mu_i(t) = \mu_{R,i}(t) + j\mu_{I,i}(t)$, where the subscripts R and I affected to a complex number denote respectively the real part and the imaginary part of the corresponding complex number. Therefore, the processes $\xi_i(t) = |\mu_i(t)|$ and $\varsigma_i(t) = |\mu_i(t) + m_i(t)|$ describe Rayleigh fading and Rician fading channels, respectively. With the assumption of flat fading, the total signal received at the relay node is given by

$$y(t) = \Big[\mu_1(t)+m_1(t)+\alpha\Big(\mu_2(t)+m_2(t)\Big)\Big]x_1(t)+n_R(t) \tag{2}$$

where $n_R(t)$ is an additive white Gaussian noise (AWGN) process. Let

$$\chi(t) = \mu_1(t) + m_1(t) + \alpha\Big(\mu_2(t) + m_2(t)\Big) \tag{3a}$$
$$= \mu(t) + m(t) \tag{3b}$$

where

$$\mu(t) = \mu_1(t) + \alpha\mu_2(t) \quad \text{and} \quad m(t) = m_1(t) + \alpha m_2(t) \tag{4}$$

are the process that describes the fading of the interference channel, and the LOS component in the EIC channel, respectively. Note that in the expression of $\mu(t)$ in (4), α acts as a scaling factor for the variance of $\mu_2(t)$ such that the corresponding process $\mu_\alpha(t)$,

$$\mu_\alpha(t) := \alpha\mu_2(t) \tag{5a}$$
$$= \Big(|\alpha|cos(\theta)\mu_{R,2} - |\alpha|sin(\theta)\mu_{I,2}\Big) + j\Big(|\alpha|sin(\theta)\mu_{R,2} + |\alpha|cos(\theta)\mu_{I,2}\Big), \tag{5b}$$

is a zero-mean complex Gaussian process of variance $2|\alpha|^2\sigma_2^2$. Since the sum of two complex Gaussian random variables is itself a complex Gaussian random variable with mean equal to the sum of means and variance equal to the sum of variances [5], we deduce that $\mu(t)$ is itself a zero-mean complex Gaussian process with variance $2\sigma^2 = 2\Big(\sigma_1^2 + |\alpha|^2\sigma_2^2\Big)$. Given that $\mu(t) = \mu_R(t) + j\mu_I(t)$, with

$$\mu_R(t) = \mu_{R,1}(t) + |\alpha|cos(\theta)\mu_{R,2}(t) - |\alpha|sin(\theta)\mu_{I,2}(t)$$
$$\mu_I(t) = \mu_{I,1}(t) + |\alpha|sin(\theta)\mu_{R,2}(t) + |\alpha|cos(\theta)\mu_{I,2}(t),$$

it appears that $\mu_R(t)$ and $\mu_I(t)$ are both zero-mean real Gaussian random variables with identical variance $\sigma^2 = \sigma_1^2 + |\alpha|^2\sigma_2^2$. In the other hand, the modulus $\rho(t) := |m(t)|$ of the LOS component in the EIC channel can be expressed as

$$\rho^2(t) = \rho_1^2 + |\alpha|^2\rho_2^2 + 2|\alpha|\rho_1\rho_2 cos\Big[2\pi(f_{\rho_2} - f_{\rho_1})t + (\theta_{\rho_2} - \theta_{\rho_1}) + \theta\Big]. \tag{7}$$

A. PDF of the First Stage Process

Throughout the paper, the process that describes the fading in the EIC channel will be referred to as the *EIC process* or the *first stage process*. Let us consider the following cases:

1) Case 1: $m_1(t) = m_2(t) = 0$: In this case, the LOS component, $m(t)$, in the EIC channel does not exist. The process $\chi(t)$ that describes the fading in the EIC channel is reduced to the scattered component $\mu(t)$. Therefore, the EIC process $\Xi(t) = |\chi(t)|$ is a Rayleigh process with parameter $\sigma^2 = \sigma_1^2 + |\alpha|^2\sigma_2^2$ and its PDF $p_\Xi(z)$ is given by [6]

$$p_\Xi(z) = \frac{z}{\sigma^2} e^{-\frac{1}{2\sigma^2}z^2}, \quad z \geq 0 \tag{8}$$

2) Case 2: $m_1(t) \neq 0$ and $m_2(t) \neq 0$: The EIC process in this case is a Rice process of LOS component modulus given in (7). Its PDF $p_\Xi(z,t)$ is given by [6]

$$p_\Xi(z,t) = \frac{z}{\sigma^2} e^{-\frac{1}{2\sigma^2}(z^2+\rho^2(t))} I_0\Big(\frac{z\rho(t)}{\sigma^2}\Big), \quad z \geq 0 \tag{9}$$

where $I_0(.)$ is the modified Bessel function of the first kind and zeroth order [7]. It is worth noting that the time dependence of $p_\Xi(z,t)$ is due to the fact the LOS component modulus of the process $\Xi(t)$ (see (7)) is time-varying in this case. However, this change in time can be avoided when the two transmitters in both sides of the relay node move such that they have the same Doppler frequency, i.e., $f_{\rho_1} = f_{\rho_2}$ which is an assumption considered in this work for simplicity. Therefore, (7) becomes

$$\rho = \sqrt{\rho_1^2 + |\alpha|^2\rho_2^2 + |\alpha|\rho_1\rho_2 cos\Big[(\theta_{\rho_2} - \theta_{\rho_1}) + \theta\Big]} \tag{10}$$

3) Case 3: $m_1(t) = 0$ and $m_2(t) \neq 0$ (resp. Case 4: $m_1(t) \neq 0$ and $m_2(t) = 0$): In this case, the modulus of the LOS component of the EIC process is given by

$$\rho = |\alpha|\rho_2. \quad \text{(resp.} \quad \rho = \rho_1.) \tag{11}$$

The EIC process is thus a Rice process with LOS component magnitude given in (11). Its PDF $p_\Xi(z)$ is given by [7]

$$p_\Xi(z) = \frac{z}{\sigma^2} e^{-\frac{1}{2\sigma^2}(z^2+|\alpha|^2\rho_2^2)} I_0\Big(\frac{z|\alpha|\rho_2}{\sigma^2}\Big), \quad z \geq 0 \tag{12}$$

$$\Big(\text{resp} \quad p_\Xi(z) = \frac{z}{\sigma^2} e^{-\frac{1}{2\sigma^2}(z^2+\rho_1^2)} I_0\Big(\frac{z\rho_1}{\sigma^2}\Big) \quad \Big), \quad z \geq 0 \tag{13}$$

B. Variance and Mean Value of the First Stage Process

Since the variance and the mean value summarize well the information provided by the PDF, let us make their derivations. Two main cases are discussed, depending on the nature of the EIC process $\Xi(t)$.

1) $\Xi(t)$ is a Rayleigh process: This is encountered in Case 1. The mean value of the EIC process is given by [8]

$$E\{\Xi(t)\} := \int_{-\infty}^{\infty} z p_\Xi(z) dz \tag{14}$$

$$= \sigma\sqrt{\pi/2} \tag{15}$$

where

$$\sigma = \sqrt{\sigma_1^2 + |\alpha|^2\sigma_2^2}. \tag{16}$$

The difference of the mean power $E\{\Xi^2(t)\}$ and the squared mean value $\Big[E\{\Xi(t)\}\Big]^2$ of the EIC process defines its variance $Var\{\Xi(t)\}$ [8]. That is,

$$Var\{\Xi(t)\} := E\{\Xi^2(t)\} - \Big[E\{\Xi(t)\}\Big]^2 \tag{17}$$

$$= (2 - \pi/2)\sigma^2 \tag{18}$$

with σ given in (16).

2) $\Xi(t)$ is a Rice process: We are in presence of Case 2, Case 3 and Case 4. Given the definition of the expected value in (14), and considering the derivation for the case of a Rice process in [8], the expected value of the EIC process when it is a Rice process is given by

$$E\{\Xi(t)\} = \sigma\frac{\sqrt{\pi}}{2} {}_1F_1\Big(-\frac{1}{2}; 1; -r\Big) \tag{19}$$

where

$${}_1F_1\Big(-\frac{1}{2}; 1 : -r\Big) = \Big[(1+r)I_0\Big(\frac{r}{2}\Big) + rI_1\Big(\frac{r}{2}\Big)\Big]e^{-r/2} \tag{20}$$

is the hypergeometric function [9], and $I_0(.)$ and $I_1(.)$ are the zeroth and the first modified Bessel functions of the first kind, respectively; $r = \frac{\rho^2}{2\sigma^2}$, where σ is given in (16), and the expressions of ρ are given in (10) for Case 2 and (11) for Case 3 and Case 4, accordingly. In addition, given the mean power derivation of a Rice process in [10], it follows that

$$E\{\Xi^2(t)\} = 2\sigma^2 + \rho. \tag{21}$$

Hence, substituting (19) and (21) into (17), we obtain

$$Var\{\Xi(t)\} = 2\sigma^2 + \rho - \Big\{\sigma\frac{\sqrt{\pi}}{2} {}_1F_1\Big(-\frac{1}{2}; 1; -r\Big)\Big\}^2. \tag{22}$$

IV. Statistical Modeling of the Second Stage

In the second stage, the relay node amplifies the received signal by a factor β and then retransmits the resulting signal. Let $s_R(t)$ be the signal transmitted by the relay node.

$$s_R(t) = \beta y(t) \tag{23}$$

where $y(t)$ is given in (2). Let $y_{S_i}(t)$, for $i = 1, 2.$, be the signal received at the sink (S_i). With the assumption of flat fading, $y_{S_i}(t)$ is given by

$$y_{S_i}(t) = \beta\Big(\mu_i(t) + m_i(t)\Big)y(t) + n_{S_i}(t) \tag{24}$$

with $n_{S_i}(t)$ being an AWGN process at the sink (S_i). Substituting (2) and (3b) into (24), the signal received at the sink (S_i) is thus expressed as

$$y_{S_i}(t) = \beta\Big(\mu_i(t) + m_i(t)\Big)\Big(\mu(t) + m(t)\Big)x_1(t) + \beta\Big(\mu_i(t) + m_i(t)\Big)n_R(t) + n_{S_i}(t). \tag{25}$$

A. PDF of the Second Stage Process

Let us denote by

$$\Upsilon_i(t) = \Big|\beta\Big(\mu_i(t) + m_i(t)\Big)\Big(\mu(t) + m(t)\Big)\Big| \tag{26}$$

the process describing the amplitude variation of the received signal envelope at the second stage. Considering the properties of the modulus, the second stage process is the product of two Rice processes: $\varsigma_{\beta_i}(t) = \Big|\beta\Big(\mu_i(t) + m_i(t)\Big)\Big|$ and $\Xi(t) =$

$\left|\mu(t)+m(t)\right|$ of parameters $\sigma_{\beta,i}^2 = \beta^2\sigma_i^2$ and $\sigma^2 = \sigma_1^2 + |\alpha|^2\sigma_2^2$, respectively.

Hence, $\Upsilon_i(t) = \varsigma_{\beta_i}(t)\Xi(t)$ is a double Rice process, whose PDF is given by [11]

$$p_{\Upsilon_i}(z,t) = \frac{z}{\beta^2\sigma_i^2\sigma^2}\int_0^\infty \frac{1}{y} e^{-\frac{(z/y)^2+\beta^2\rho_i^2}{2\beta^2\sigma_i^2}} e^{-\frac{y^2+\rho^2(t)}{2\sigma^2}} I_0\Big(\frac{z\rho_i}{y\beta\sigma_i^2}\Big) I_0\Big(\frac{y\rho(t)}{\sigma^2}\Big)dy, \quad z \geq 0 \tag{27}$$

with $\rho(t)$ expressed in (7). Note that this process occurs when there exists a LOS component in the second stage with at least one LOS component in the first stage.

When no LOS component exists both in the first stage and in the second stage, the process $\Upsilon_i(t)$ is reduced to a double Rayleigh process, whose PDF is given by [7]

$$p_{\Upsilon_i(t)} = \frac{z}{\beta^2\sigma_i^2\sigma^2} K_0\Big(\frac{z}{\beta\sigma_i\sigma}\Big), \quad z \geq 0, \tag{28}$$

where $K_0(.)$ is the modified Bessel function of the second kind and zeroth order [7].

For the other cases, the process $\Upsilon_i(t)$ is a (Rayleigh$\times$Rice) process, whose PDF can be derived from (27) accordingly.

B. Variance and Mean Value of the Second Stage Process

Since from the double Rice process expressions, we may derive the expressions of the double Rayleigh process and the (Rayleigh$\times$Rice) process, we restrict the mean value and the variance expressions of the second stage process to the double Rice process case. Therefore, the mean value $E\{\Upsilon_i(t)\}$ and the variance $Var\{\Upsilon_i(t)\}$ of the second stage process are given respectively by [11]

$$E\{\Upsilon_i(t)\} = \frac{\sigma\beta\sigma_i\pi}{2} {}_1F_1\Big(-\frac{1}{2};1;-\frac{\rho_i^2}{2\sigma_i^2}\Big) \times {}_1F_1\Big(-\frac{1}{2};1;-\frac{\rho^2}{2\sigma^2}\Big) \tag{29}$$

$$Var\{\Upsilon_i(t)\} = 4\sigma^2\beta^2\sigma_i^2\Big(1+\frac{\rho_i^2}{2\sigma_i^2}\Big)\Big(1+\frac{\rho^2}{2\sigma^2}\Big) - \Big(\sigma\beta\sigma_i\frac{\pi}{2}\Big)^2 \times \Big\{{}_1F_1\Big(-\frac{1}{2};1;-\frac{\rho_i^2}{2\sigma_i^2}\Big){}_1F_1\Big(-\frac{1}{2};1;-\frac{\rho^2}{2\sigma^2}\Big)\Big\}^2 \tag{30}$$

V. Simulation and Discussion

In this section, we will evaluate, through simulations, the correctness of the analytical expressions derived, and we discuss the results with regard to some requirements in PLNC.

A. The Deterministic Simulator

The simulations performed are based on the concept of deterministic sum-of-sinusoids (SOS) channel modeling [12]. It has been shown [13] that in general, deterministic SOS simulators have a better efficiency compared to non-ergodic stochastic SOS channels simulators. Briefly, let us review the deterministic SOS simulator used to generate the K multiple mutually uncorrelated complex Gaussian processes that make up the statistical processes in both the first stage and the second stage of the PLNC. The complex deterministic Gaussian process $\tilde{\mu}^{(k)}(t) = \tilde{\mu}_1^{(k)}(t) + j\tilde{\mu}_2^{(k)}(t), \quad (k = 1,...,K)$, that models the distortion of the channel caused by its time-varying nature, is given such that its in-phase component $\tilde{\mu}_1^{(k)}(t)$ and quadrature component $\tilde{\mu}_2^{(k)}(t)$ are expressed by [14]

$$\tilde{\mu}_i^{(k)}(t) = c_{i,n}^{(k)} \sum_{n=1}^{N_i} cos(2\pi f_{i,n}^{(k)} t + \theta_{i,n}^{(k)}) \quad \text{for} \quad i = 1,2. \tag{31}$$

where $c_{i,n}^{(k)}$, $\theta_{i,n}^{(k)}$ and $f_{i,n}^{(k)}$ are respectively the Doppler coefficients, the phases and the discrete Doppler frequencies of the n^{th} sinusoid of the in-phase and quadrature components. And N_i is the number of sinusoids. It has been shown [14] that values of $N_i \geq 7$ are sufficient to approximate the deterministic distribution process $|\tilde{\mu}^{(k)}(t)|$ very close to the theoretical Rayleigh distribution. In this work, we set $N_1 = N_2 = 30$. The phases $\theta_{i,n}^{(k)}$ are considered as outputs of independent and identically distributed random variables $\Theta_{i,n}^{(k)}$, each having a uniform distribution over the interval $(0, 2\pi]$. The Doppler coefficients are given by $c_{i,n}^{(k)} = \sqrt{2/N_i}$, and the discrete Doppler frequencies $f_{i,n}^{(k)}$ are computed according to the generalized method of exact Doppler spread (GMEDS$_q$) [12]:

$$f_{i,n}^{(k)} = f_{\max} cos\Big[\frac{q\pi}{2N_i}\Big(n-\frac{1}{2}\Big) + \alpha_{i,0}^{(k)}\Big] \tag{32}$$

where $f_{\max}$ is the maximum Doppler frequency and $\alpha_{i,0}^{(k)}$ is the angle of rotation depending on the value of $q = \{0, 1, 2\}$. In this work, we have chosen $q = 1$ so that

$$\alpha_{i,0}^{(k)} = (-1)^{i-1}\frac{\pi}{4N_i}\frac{k}{K+2}. \tag{33}$$

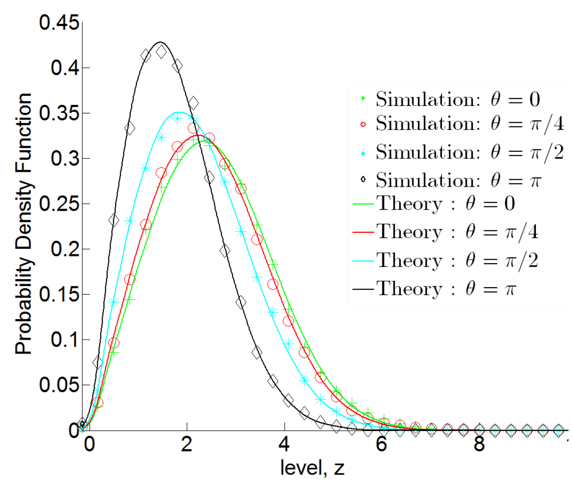

Fig. 2. Variation with respect to θ of the first stage process PDF in presence of the LOS components for both channel (1) and channel (2).

B. Discussion

In addition to the rigorous mathematical approach, the good fitting of both the analytical and simulation results, as depicted in Fig. 2, confirms the accuracy of the analytical expressions of the statistics. It is worth noting that due to lack of space, we present only few results in our analysis. We consider the case where the first stage process $\Xi(t)$ is a Rice process, and the second stage process $\Upsilon_i(t)$ is a double Rice process.

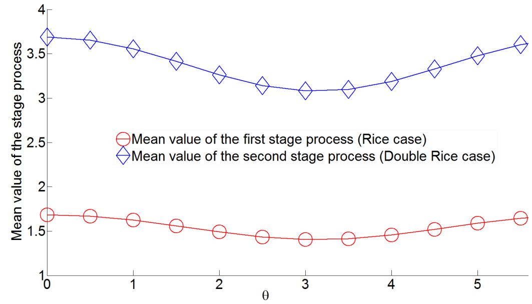

Fig. 3. Comparison of the mean values of both the first stage and second stage processes with respect to θ.

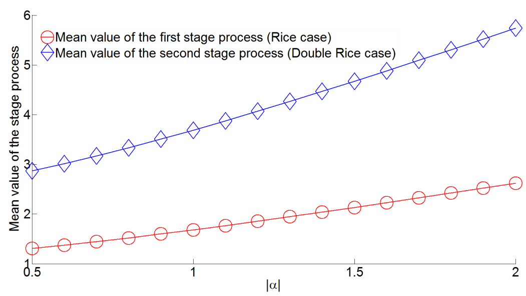

Fig. 4. Comparison of the mean values of both the first stage and second stage processes with respect to $|\alpha|$.

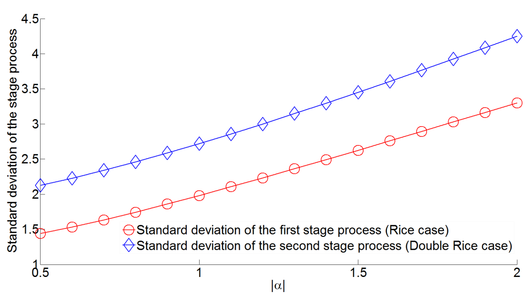

Fig. 5. Comparison of the standard deviations of both the first stage and second stage processes with respect to $|\alpha|$.

The effect of the variation of $|\alpha|$ and θ on the statistics of the stages processes is more explicitly shown in Fig. 3–Fig. 6. In Fig. 3 and Fig. 4, are presented respectively the mean value of the stages processes, for different values of θ and $|\alpha|$. It can be observed that the mean value of the second stage process is higher than the one of the first stage. Same observations are noticed in Fig. 5 and Fig. 6 which present respectively the effect of $|\alpha|$ and θ on the standard deviation (square root of the variance) of the stage processes. In Fig. 4 and Fig. 5, one may remark that any increase of $|\alpha|$ has as effect the increase of both the standard deviation and the mean value of the processes. A different observation is done when

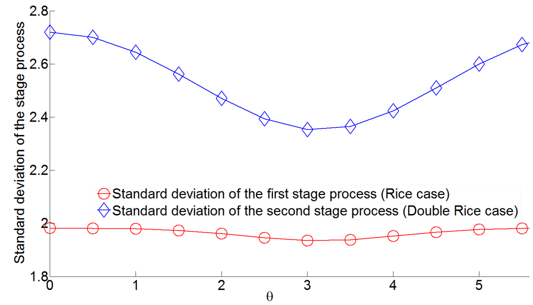

Fig. 6. Comparison of the standard deviations of both the first stage and second stage processes with respect to θ.

evaluating the effect of θ on both the standard deviation and the mean value of the stage processes. Indeed, we observe that there exists a minimum for both the standard deviation and the mean value around the value $\theta = \pi$. This is more noticeable in the second stage compared to the first stage as shown in Fig. 3 and Fig. 6.

VI. Conclusion

In this paper, we have studied the statistics of the processes that describe the fading of the received signals envelope magnitude in PLNC. The investigation was based on a simple interference channel model that captures both the phase and magnitude differences between the transmissions in both sides of the relay node in a TWRC. The theoretical analysis thus presented is useful for researchers and designers of PLNC techniques for more realistic communication environments.

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