

# M-PNC: Multi-Hop Physical Layer Network Coding for Shared Paths in Vehicular Networks

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**Abstract** In wireless networks, spectral efficiency is an important issue due to limited available bandwidth. Traditionally, when a receiver collects simultaneous transmissions, packets collide and the much meaningful information cannot be retrieved. Thus, through suitable scheduling, simultaneous transmissions to the receiver are avoided. However, in physical layer network coding, simultaneous transmissions are allowed and the combined signal is equivalent of XOR operation on air. A path from a source to a destination in unicast routing in VANET is called a flow; multiple flows may have common road segments. There will be heavy contention in such shared road segments resulting in lower throughput, lower packet delivery ratio and higher end to end delay. In this paper, we propose a multihop physical layer network coding structure to be used by multiple flows of unicast routing paths. We analyze throughput, end-to-end delay and coding gain with respect to number of routing flows and number of forwarders involved in network coding.

**Keywords**— Analog Signal, Network Coding, Power Control, Spectrum Access

## I. INTRODUCTION

With the advancement of wireless technology and development in the automobile industry, the current road transportation involves intelligent vehicles. Over the last few years, Vehicular ad Hoc networks (VANETs) have drawn considerable attention from research community because of the promising services and applications provided through the cooperation of equipped vehicles. The major goal of VANETs is to increase the safety of passengers and enhance the driving experience by providing services, such as collision warning, lane change assistance, intelligent navigation and road traffic control. Apart from safety applications, infotainment applications, such as advertisement, marketing, and business of services and products on wheels appear to be very lucrative and promising in terms of commerce and research.

Unicast routing is widely used as a data delivery model in VANETs for both safety and infotainment applications. Generally, the content in VANETs needs to be delivered to a destination few hops away from the source. As a result, design of multi-hop unicast routing has attracted significant attention from research community. In unicast routing, a routing path is determined between source and destination. The criterion for determining the path is however different from one routing protocol to another based on the application's requirements. The most common criteria are shortest hop-count and highest connectivity. When several unicast flows (i.e. source-destination pairs) exist in VANETs, the routing paths are likely to overlap. For example, a highly connected road segment is shared among multiple unicast flows. As a consequence, severe contention results in the shared region.

Packets are dropped due to increased collisions and delivery is delayed at destinations; which eventually leads to a decrease in throughput.

Network coding [1] has been studied extensively in wireless networks to improve throughput of unicast routing. Traditionally, network coding is applied to packets that belong to different unicast flows. The network coding of this kind is referred to as digital network coding (DNC) or packet level network coding. The intermediate relay nodes perform XOR operation on multiple input packets and produce multiple output coded packets. The throughput is further enhanced by the use of physical layer network coding (PNC) which is applied to physical layer signals rather than packets. In wireless networks, interferences are avoided by scheduling transmissions in different time slots. With PNC, interferences from concurrent transmissions are exploited to enhance the throughput of wireless networks. PNC is achieved through two schemes: (1) AF (Amplify and Forward) [12] where the relay node amplifies the received interfered signals and forwards to the transmitters; and (2) DF (Decode and Forward) [5][6] where the relay node first decodes the interfered signals, re-encodes and then forwards to the transmitters.

Usually, a routing protocol determines a path to forward unicast messages. In a city, multiple sources may generate packets at the same time. As busy roads are considered to be most connected paths, some of the routing paths may have common road segments. This is the ideal case for network coding. To exploit best out of such a situation, in this paper, we propose a DF based physical layer network coding scheme to increase throughput in vehicular networks. In our scheme, we define three units: Physical Layer Network Coding unit (PNC Unit), Input unit and Output unit. The PNC unit is the shared path region where multihop physical layer network coding is conducted, the input unit provides the input data and packets are uncoded in output unit.

The rest of the paper is organized as follows. A brief discussion of related work is presented in Section 2. Section 3 presents our motivations. Section 4 describes the proposed mechanism. Section 5 presents performance evaluation. Finally, Section 6 concludes the paper.

## II. RELATED WORK

Network coding (i.e. DNC) was first proposed for wired networks (e.g. butterfly network scenario) in [1]. The authors show that multicast capacity is achieved by allowing the routers to perform network coding operation on different forwarded packets. The network coding operation is basically a bit-wise XOR operation which can also be viewed as linear

coding over the finite field Galois Field GF ( $2^n$ ). Later, in [2], Li et al. show that, for multicast traffic, it is possible to achieve the maximum capacity bounds using linear codes while coding and decoding can be performed in polynomial time. In [3], a distributed random linear network coding is proposed where routers pick random coefficients from a finite field without knowing network topology a-priori. The authors proved that with a sufficiently large field size, nodes can decode the XOR coded packets with high probability. The practical use of network coding for wireless networks have been investigated thoroughly in the literature [4][7]. The network coding we discussed so far falls under the category of DNC. In recent years, another form of network coding, termed as physical layer network coding (PNC) or analog network coding (ANC), has emerged. In contrast to DNC which operates on different packets, PNC uses network coding operation on physical layer signals.

Some theoretical contributions [9][10] show that network coding on interfering signals doubles the capacity of the canonical 2-way relay network. The throughput capacity of random wireless networks with PNC is studied in [14]. The authors show that although PNC does not change the scaling laws, it improves throughput by a fixed factor compared to relaying with DNC and relaying without network coding. Some recent works [8][11][13] study the outage probability and BERs in case of PNC. In [5] and [6], modulation and demodulation techniques are proposed for PNC. In [6], the superposition of electromagnetic (EM) signals is mapped to GF ( $2^n$ ) additions of digital bit streams. The mapping allows interferences to be perceived as XOR operation of two transmitted signals. PNC assumes that EM signals are received with same phase and amplitude using appropriate carrier-phase and symbol time synchronization schemes. Analog network coding (ANC) is proposed in [12]. The authors in [12] argue that it is unlikely for two signals to arrive in same phase at the router and suffer the same amount of attenuation over the wireless channel. In fact, they enforce lack of synchronization by inserting random delays before a transmission. In ANC, the superposed signal is not decoded; instead the routers amplify and forward the superposed signal. The algorithm in ANC is developed for only one-hop transmissions only. However, for multi-hop networks, complex mechanisms are needed to schedule concurrent packet transmissions; this is not discussed in [12]. Moreover, the decoding operation becomes difficult when the superposed signal consists of more than two EM signals.

### III. BACKGROUND AND MOTIVATION

In PNC as shown in Fig. 1, the number of slots is reduced to two, whereas in traditional forwarding, the number of transmissions will be four. Basically, the superposition nature of electromagnetic waves is exploited in PNC. Thus, the coding gain of physical layer network coding over traditional transmission is 100%. Fig.1 shows that nodes S1 and S2 send messages A and B simultaneously. In traditional forwarding, the relay node receives one of the messages or discards both messages. However, using physical layer network coding, packets from both directions will be received simultaneously. Thereafter, the relay node rebroadcasts the superposed signal which is  $A \oplus B$ . As shown in Fig 1, node D2 (incidentally it is S1 as well) can extract B by  $\ominus$  operation of  $(A \oplus B)$  and A.

Similarly, node D1 can extract A. On busy roads, some of the vehicles always wait for red signal to be green to make a go in a round robin fashion (see Fig. 2). These busy roads allow for huge amount of data to be forwarded as these road segments share routes with multiple routing flows. Vehicular networks make use of IEEE 802.11p which supports multiple channels namely control channel and service channels. Using control channel message exchanges, one of the vehicles can be selected as the position bearer of an intersection. Again, using control message exchanges, a new vehicle can be chosen as the position bearer once the old position bearer makes a move. There is always a node to take care of routing packets in an intersection as a static node though virtual. This makes an interesting opportunity for network coding. A multi-hop physical layer network coding would be ideal for such situations. In the literature [4][6] multi-hop physical layer network coding is conducted for a path (e.g. node A to node B) and all the source and destination nodes are part of that path (i.e. AB is the superset of all paths  $S_i-D_i$  pairs, where  $S_i$  is the source node and  $D_i$  is the destination node). Alternatively, the total path from the source to the destination is part of the Network coding. These kinds of scenarios are highly unlikely in real world. Thus, we consider scenarios where multiple routes shares parts of their routes with each other. It is quite beneficial in terms of throughput for routing messages when data and acknowledgements share the common path in different directions. Here, messages from opposite directions exploit the collision instances. In Fig. 3, multiple routes share part of their paths; this scenario cannot be addressed in existing methods while it can in our proposed solution.

## IV. PROPOSED SCHEME

### A. PNC Basics

To illustrate PNC, let us consider the topology in Fig. 1. There are two unicast flows in which data packets are relayed through relay node  $N_2$ . Node  $N_1$  and node  $N_3$  act as source and destination to each other. Let us assume that BPSK modulation is used to transmit the signal. We also assume symbol-level time and carrier-phase synchronization. By adjusting transmit power; packets from  $N_1$  and  $N_3$  arrive at  $N_2$  exactly at the same time and with same amplitude. The combined band pass signal received by  $N_2$  during one symbol period is given by:

$$\begin{aligned} r_2(t) &= s_1(t) + s_3(t) = a_1 \cos(\omega t) + a_3 \cos(\omega t) \\ &= (a_1 + a_3) \cos(\omega t) \end{aligned} \quad (1)$$

where  $s_1(t)$  and  $s_3(t)$  denote the bandpass signal transmitted by  $N_1$  and  $N_3$  respectively,  $r_2(t)$  is the band pass signal received by  $N_2$  during one symbol period,  $a_1$  and  $a_3$  denote the BPSK modulated information bit of  $N_1$  and  $N_3$  respectively, and  $\omega$  is the carrier frequency. Then,  $N_2$  will obtain baseband signal  $a_1 + a_3$ . Note that  $N_2$  cannot decode the individual information transmitted by  $N_1$  and  $N_3$ , that is,  $a_1$  and  $a_3$ , from the combined signal in  $a_1 + a_3$ . But, with the help of modulation and demodulation schemes,  $N_2$  can transmit the combined signal which is used by  $N_1$  and  $N_3$  to decode each other's signal. To achieve this, we use the PNC mapping scheme proposed in [6]; the objective of this mapping is to obtain an equivalence of GF( $2^n$ ) summation of bits from  $N_1$  and  $N_3$  at the physical layer.

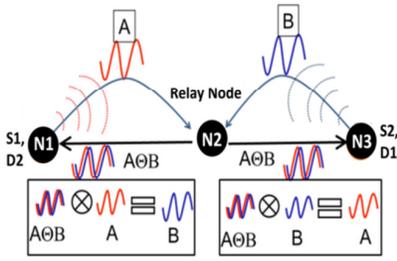


Fig. 1 PNC in 1-Hop

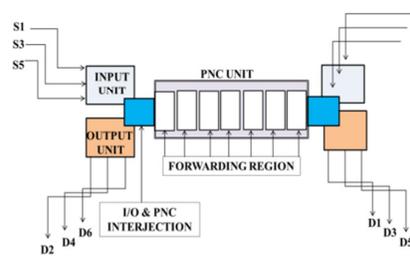


Fig. 2. Multi-hop PNC Structure

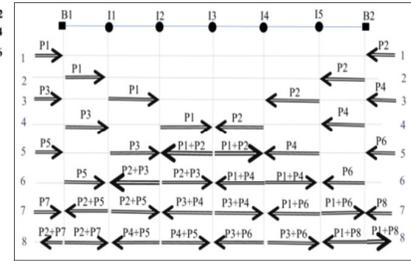


Fig. 3. PNC for Shared Regions

According to PNC mapping, the data of source node is represented as  $s_j \in \{0, 1\}$ , where  $s_j$  is a variable representing the data bit of  $N_j$  and  $a_j \in \{-1, 1\}$  is a variable representing the BPSK modulated bit of  $s_j$  such that  $a_j = 2s_j - 1$ . At the relay node, a mapping scheme is used to convert the combined signal into binary bits. A strong energy signal (i.e. signal with higher value of absolute amplitude) is converted to binary 0 and a weak energy signal (i.e. signal with lower value of absolute amplitude) is converted to binary 1. For example, the combined signal with  $|a_1 + a_3| = 2$  and with  $|a_1 + a_3| = 0$  is mapped into binary 0 and binary 1 respectively. Using the above mapping, the information bit of the combined signal is represented as the XOR operation of the signals  $s_1$  and  $s_3$ . After obtaining  $s_3$ , relay node  $N_2$  then transmits  $s_2 = a_2 \cos(\omega t)$ .

### B. Protocol in a Nutshell

The proposed PNC scheme allows multiple unicast flows to share a road segment without degradation in throughput. In our approach, we exploit and extend the benefits of PNC by allowing simultaneous transmissions. We assume that one unicast flow shares part of its routing path with another unicast flow. We adopt multi-hop physical layer network coding approach which allows simultaneous transmissions in multiple hops. We design a multi-hop PNC structure which consists of three components: PNC unit, Input Gate and Output Gate. A PNC unit consists of a group of forwarding nodes (i.e. part of shared routing path) that belong to multiple routing paths. The forwarding nodes on either end of the PNC unit are termed as border gates. The purpose of input gate is to provide uncoded packets to the border gate. On the other hand, the purpose of output gate is to extract the information bits from PNC coded signals. This paper proposes a multi-hop physical layer network coding scheme for multiple communication links. In our scheme, we divided the entire setup in three segments such as PNC Unit, Input Unit and Output Unit.

1) *PNC unit*: In single hop PNC as shown in Fig. 1, when EM signals from node  $N_1$  (i.e.  $s_1$ ) and node  $N_3$  (i.e.  $s_3$ ) are received at relay node  $N_2$  at the same time, the superposed signal  $r_2$  given in Eq. (1) is obtained. The superposed signal is then mapped to signal  $s_2$  which is the result of XOR on air. When  $N_1$  and  $N_2$  receive  $s_1 \oplus s_3$ , they can extract the other signal XORing  $s_2$  with their own signal. However, in case of multi hop physical layer network coding, the process must happen at every alternate hop. As shown Fig. 3, when packets P1 and P2 arrive at node I3 simultaneously (time slot 4), the XOR coded packet  $P1 \oplus P2$  is obtained. When  $P1 \oplus P2$  is forwarded by I3 at time slot 5, packets P3 and P4 are

forwarded to node I2 and node I4 respectively. However, node I2 and node I4 have packets P1 and P2 respectively which were available to them earlier. Thus, the extracted packet could be:

$$P1 \oplus ((P1 \oplus P2) \oplus P3) = P2 \oplus P3 \quad (2)$$

$$P2 \oplus ((P1 \oplus P2) \oplus P4) = P1 \oplus P4 \quad (3)$$

In the next instance (i.e. time slot 6),  $P2 \oplus P3$  and  $P1 \oplus P4$  is sent to I3 and the extracted message is  $(P2 \oplus P3) \oplus (P1 \oplus P4) \oplus (P1 \oplus P2) = P3 \oplus P4$ . Thus, in time slot 7,  $P3 \oplus P4$  is forwarded by I3. Data packets are received from both ends (e.g. Border Node B1 and Border Node B2) of PNC in alternate intervals. Let us look at B1 for all packets those arrive or leave B1; at interval 7, B1 collects P7 as input (i.e. from left) and P2 as output. Here, P2 is not an isolated packet, rather it is coded with P5 (i.e.  $P2 \oplus P5$ ). However, B1 can extract  $P2 \oplus P7$  out of P7 and  $(P2 \oplus P5)$  as it has P5 packet stored previously.

Table-I

M-PNC
<pre> Initialize() {Sq=(Fid) Mod 2;// Fid is Forwarder Id , T=SetTimer(Sq); RMessage=NULL, MForward=NULL; FR=False; //It becomes true on receiving 1st message }  ProcedurePNC() {   If ( T==1){     /* T =1 is the slot to receive Message. If a forwarder is in     receive mode (i.e. T=1), the neighboring forwarder must be in     sending mode. It changes alternatively.*/     RMessage=ReceiveMessage();     /*RMessage is the latest received message */     If(RMessage != NULL)     {       If(FR==False){         FR=True; MForward=RMessage;         /* MForward is the message ready to be forwarded         in next timeslot */       }       Else {         MForward=(RMessage) XOR (MForward)         /*XOR current message and last forwarded message; this         message is forwarded in next time slot */       }     }     Else{ Initialize();}   }Else   {     If(RMessage!=NULL) // sends message MForward     Send(MForward);   }   UpdateTimer(T); // This toggles T between 0 and 1 } </pre>

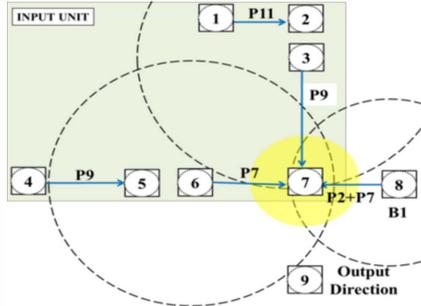


Fig. 4 Input Unit in Interval J.

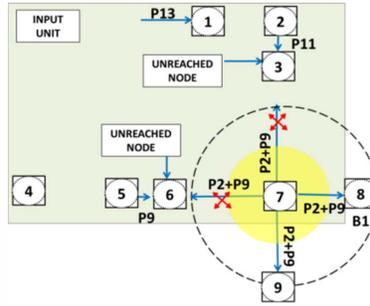


Fig 5 Input Unit in Interval J+1

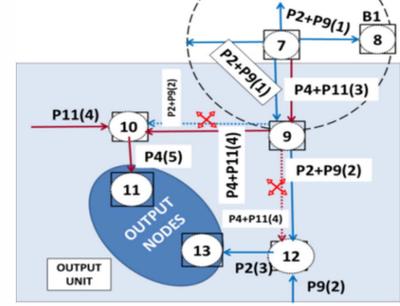


Fig 6 Output Unit

The direction of P7 is towards right and direction of P2 is towards left. It is perfectly fine to send  $P2 \oplus P7$  towards right. However, when  $P2 \oplus P7$  is sent towards left, we should be able to extract P2 somewhere to send to the desired destination. Therefore, an output module is needed for this purpose. Similarly, in the next instance B1 should receive non-coded packets. Thus an input module is essential for this purpose. Fig. 2 shows INPUT unit, OUTPUT unit and PNC unit for such purposes. Fig. 3 depicts the details of PNC unit; Fig 4 and Fig 5 explain the details of the Input unit; and Fig 6 is the elaboration of the output unit. The pseudo-code of the distributed forwarding algorithm is shown in Table-I. We assume all the forwarders (i.e. Border Nodes: B1 and B2, Inter mediate Nodes: I1, I2, I3, I4, and I5. Other Nodes: 1,2,3, so on) synchronized to two time slots ( slot 0 and slot 1). From the examples given (i.e. Fig. 3, Fig. 4, Fig. 5, and Fig. 6) the forwarding nodes start with following time slots.

- Time Slot '0'- I1, I3, I5, 2, 5, 7, 10, 12,
- Time Slot '1'- B1, B2, 12, I4, 1, 3, 4, 6, 8, 9,

2) *Input unit*: Node B1 (in Fig 3, it is also node 8 in Fig. 4 and Fig. 5) acts as the interface between the bidirectional flow. In one time slot, it receives the packet from opposite directions and in the other time slot it forwards the packet to both the directions (i.e. coded packet). Initially, it receives a single packet from left (e.g. P1, P3, P5) when no packet arrives from opposite direction. Later, it receives an uncoded packet from the left (e.g. P7) and coded packet from the right (e.g.  $P2 \oplus P5$ ). A coded packet  $P2 \oplus P7$  (i.e.  $P7 \oplus ((P2 \oplus P5) \oplus P5) = P2 \oplus P7$ ) is extracted from these two packets. Remember that P5 is stored in B1 which it has received during previous time slots. In the next time slot, B1 receives  $P2 \oplus P9$  from the left and  $P4 \oplus P7$  from its right. Now,  $P4 \oplus P9$  can be extracted by combining  $(P2 \oplus P9)$ ,  $(P4 \oplus P7)$  and  $(P2 \oplus P7)$ . This packet is forwarded by B1. We notice that, in saturation case, B1 receives two packets and forwards a coded bidirectional packet alternatively. In other words, B1 receives input packet to be sent towards right and receives output packet to be sent towards left in a single time slot. Thus, the input and output packets should be separated. As shown in Fig 4 new uncoded packet (P9) is sent by node 3 and coded packet ( $P2 \oplus P7$ ) is sent by node 8 to node 7. As node 7 needs to send coded packet ( $P2 \oplus P9$ ) in the next instance (e.g. Fig 5), there must be another input of P7 towards node 7 to obtain  $P4 \oplus P9$  from  $(P2 \oplus P7)$ , (P9) and (P7). Thus, we have an arrangement (Placement of

forwarding node or adjustment of transmission power) in Fig 4 and Fig 5 in which packets from node 3 and node 6 can reach node 7, but no packet can reach node 7 from node 3 and node 6.

3) *Output unit*: In Fig. 6, we illustrate the process of packet extraction from a coded packet. Node 9 chooses two paths alternatively to extract the packets (e.g. towards node 12 and node 10). Two paths are needed as coded packets are received at node 9 in alternate time slots. However, we need 4 time slots to extract an output packet from coded packet (e.g. slot 1- receives coded packet, slot-2 – receives an uncoded packet which is part of the coded packet, slot-3 forwards the extracted packet, slot-4 remains silent to allow extracted packet to move 2 hops away. ). As shown in Fig 6, the coded packet  $P2 \oplus P9$  received by node 9 is forwarded to node 12; at that time packet P9 is received by node 12. Thus, P2 can be extracted and sent to node 13. At the same time node 9 receives packet  $P4 \oplus P11$ . Therefore, the transmission power of node 12 is such that no packet is received at node 9 from node 12. When  $P4 \oplus P11$  is forwarded by node 9, node 12 should ignore all packets it receives as it is supposed to receive P2 and  $P4 \oplus P11$  which will be a complex coded packet. However, node 10 must receive this packet (i.e.  $P4 \oplus P11$ ) along with P11 packet. Here, packet P4 is extracted and forwarded to node 11.

## V. PERFORMANCE EVALUATION

### A. Throughput

1) *Analysis*: In this section, we evaluate the end-to-end flow throughput of both the DNC and PNC transmission schemes in unidirectional flows. Let us consider an unidirectional linear network with  $n$  nodes. Nodes are labeled node 1, node 2, .. node  $n$ , where node 1 and node  $n$  are the source node and the destination node, respectively. We assume that the source node has an infinite number of packets that need to be sent to the destination node. We also assume that a packet is received successfully by the destination when all the information bits are received correctly; for any erroneous packet, retransmissions occur until it is correctly received. This is done on a per-link basis.

Let  $T_h$  denotes the transmission time of an individual packet. It includes the inter-frame spaces, RTS transmission time, CTS transmission time, ACK transmission time and DATA transmission time. Let  $P_s$  denotes the probability that a receiver receives the decoded signals successfully, i.e.

without any interference from transmitters on either direction. Then, the number of failed transmissions before a decoded signal is received successfully is given as below:

$$\delta = \frac{1 - p_s}{P_s}, \text{ where } \delta \text{ is a geometrically distributed random}$$

variable. Then, the transmission time in one hop is given by:

$$T_{Delay} = T_h (1 + \delta) \quad (4)$$

If  $P_b$  denotes the bit error rate, then the packet success probability  $P_s$  is expressed as:

$$P_s = 1 - (1 - P_b)^L \quad (5)$$

where  $L$  denotes the total number of bits in a packet.

In our proposed scheme, each node can decode a packet every two time slots. In other words, destination receives a packet in every two time slots. Hence, the throughput in case of M-PNC is given by:

$$\alpha = \frac{L}{2 * T_{Delay}} \quad (6)$$

2) *Results:* We investigate throughput (i.e.  $\alpha$ ) with respect to number of hops (i.e.  $h$ ) between the relay node and interfering nodes. The signal-to-interference ratio (SIR) at a receiver is given by:

$$SIR = \frac{P_r / d^2}{2 * \sum_{i=1}^h P_r [(2 * i + 1) d]^2} \quad (7)$$

where  $d$  denotes the distance between consecutive forwarders. For BPSK Modulation, the relationship between SNR per bit  $E_b/N_o$  and bit error rate  $P_b$  is given by:

$$P_b = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{N_o}} \right) \quad (8)$$

For each value of  $h$ , SIR is obtained from Eq. (7). From SIR, SNR per bit is obtained as follows :

$$SIR = \frac{E_b}{N_o} \frac{f_B}{B} \quad (9)$$

where  $f_b$  and  $B$  denote the data rate and channel bandwidth respectively. Using  $P_b$ , the throughput is calculated as described in Eq(6). Similarly, throughput for QPSK Modulation is calculated.

### B. End to End Delay

A single node can use 1 time slot to receive and 1 time slot to forward packets. At the entry point of the shared path region (e.g. PNC UNIT entry in Fig. 3) a single node is used to receive and forward packets. As shown in Fig 3, the PNC Unit can accommodate 1 packet in 2 time slots from each direction or two packets in 2 time slots from both directions. The packets from opposite directions can be sent to the same forwarder simultaneously; thus, no additional slots wasted in waiting in queue. In such a saturated case overall delay of each flow is given as follows:

$$D = H * T_{Delay} + W_q + Q \quad (10)$$

where  $D$  is the delay of a routing flow,  $H$  is the number of hops from source to destination,  $T_{Delay}$  is the 1-hop

transmission delay,  $W_q$  is the queue delay and  $Q$  is the delay due to contention. Assuming the system uses TDMA, the contention delay will be 0 and in M-PNC the arrival and departure of the packets remain the same. Therefore, the queue delay is considered to be 0 as well. Thus,  $D$  is having delay of the order of  $n$ . In a system shown in Fig. 7, 'S' packets arrive at the rate of  $\lambda$  at A and get out of E at the rate of  $\mu$  towards the shared path regions. Let us note that packets come from Y, go to K. In A, maximum arrival could be 1 packet per 2 time slots. Similarly, average departure rate at E towards Y could be 1 packet per 4 time slots. A system cannot be in steady state if the arrival rate is bigger than the departure rate. Here the arrival rate is the double of the departure rate. Considering the system follows M/M/1 queueing model, the queue is not in steady state and the queue delay would tend to infinity at some point [15]. If the packets have to stay in queue for infinite time, they cannot reach the destination and the delay for the routing flow becomes infinity.

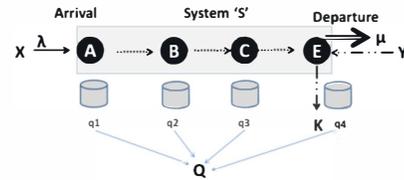


Fig. 7. Non-Physical Layer Network Coding

### C. Coding Gain

Coding gain is the number of transmissions required without using network coding over number of transmissions when network coding is used. We consider simultaneous transmissions to the same receiver as a single transmission. In Fig. 1, the total number of transmissions is two; whereas in traditional scheme (i.e. DNC), the number of transmissions would be four. Therefore, the coding gain for single hop is  $4/2 = 2$ . It shows 100% gain of PNC over DNC. However, we measure the benefit of physical layer network coding for multiple flows. With increase in number of flows, the number of hops is dynamically adjusted to support additional flows in our scheme. For the evaluation of coding gain, we keep  $M$  routing flows with each flow having  $N$  hops and they share  $H$  hops (i.e. traditional case). With M-PNC,  $H$  is increased or decreased based on  $N$ . Thus the coding gain can be expressed as follows:

$$C = \frac{M * N}{M * (N - H) + H + H_A} \quad (11)$$

where  $H_A$  denotes the number additional hops to conduct Physical Layer Network Coding (e.g. Input and Output Unit shown in Fig. 3). For better understanding of the coding gain, we choose to represent Eq(11) in graph in Fig. 10.

### D. Performance Comparison

Fig. 8 and Fig. 9 show throughput variation with respect to the number of hops between a receiver and simultaneous transmitters for BPSK modulation and QPSK modulation respectively. The throughput obtained with the proposed scheme is compared with packets relaying without network coding. The rationale behind this investigation is that vehicular networks exhibit dynamic wireless channel due to presence of buildings and static/moving obstacles.

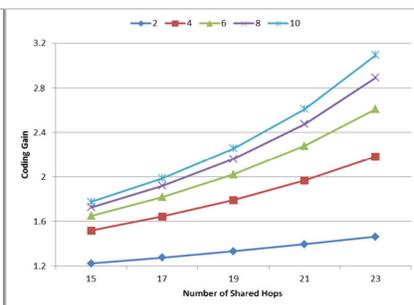
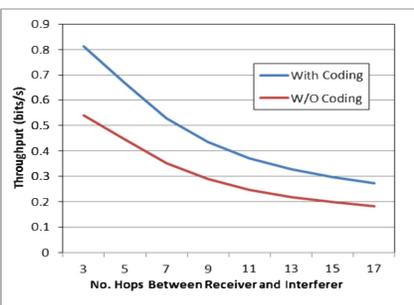
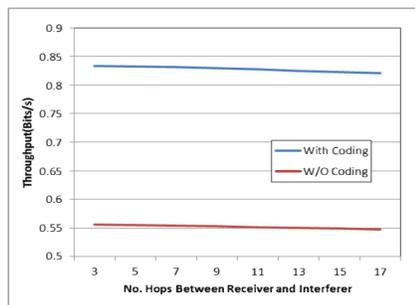


Fig. 8. Throughput (BPSK Modulation)

Fig. 9. Throughput (QPSK Modulation)

Fig. 10. Coding Gain

We, therefore, consider impact of cumulative interferences on throughput by varying the number of hops between a receiver and interfering vehicles. In Fig. 8, we observe that throughput decreases with increase in number of hops between receiver and interfering vehicles. This is due to the fact that a higher number of hops indicate higher number of interfering vehicles. As a result of cumulative interferences though small, bit error rate increases which leads to higher number of collisions and hence reduces the throughput. Because of PNC, the proposed scheme outperforms packet relaying without coding (i.e. traditional forwarding). The performance gap remains almost same at around 50 % for all values of number of hops.

Fig. 9 shows the impact of number of hops on throughput when QPSK modulation is used for data transmission. We observe that M-PNC scheme outperforms packets relaying without network coding by 50% and 49.7% for smallest and highest number of hops respectively. Both schemes experience lower throughput as compared to BPSK modulation. This is due to the fact that QPSK is less robust than BPSK resulting in an increase of the number of collisions. We observe that both schemes experience 26.5% and 66% reduction in throughput for smallest and highest number of hops respectively.

Fig. 10 shows the impact of number of flows and number of hops in shared region (i.e. PNC unit in Fig 2) keeping the flow length fixed. Let us assume that  $N=30$  (total number of hops for each flow) and  $H_A=4$  (i.e. additional hops required by input and output units). We vary  $H$  (total number of hops in PNC unit) from 15 to 23 and  $M$  (number of flows) from 2 to 10. The total gain is represented in Fig 10. Note that we consider the saturated case (i.e. at least 1 packet is available to forward in each alternate time slot from either directions). The coding gain is as high as 3.2 for  $M=10$  and  $H=23$  and the lowest is 1.2 for  $M=2$  and  $H=15$ . Thus, in multihop routing which adopt physical layer network coding would have much better throughput.

## VI. CONCLUSION

In wireless networks, spectral efficiency is an important issue due to limited available bandwidth. Traditionally, when a node receives simultaneous transmissions, packets collide and the intended packets cannot be retrieved. Packet scheduling is required to avoid simultaneous transmissions. However, in physical layer network coding, signals are strategically transmitted simultaneously to improve throughput. In our approach, we enhance the capabilities of

physical layer network coding by extending it to multi-hop. Our M-PNC scheme has three units, namely (1) PNC unit meant for physical layer network coding to be used by shared paths; (2) The input unit which sends un-coded packet to the PNC unit; and (3) The output unit which is responsible for extracting un-coded packets from coded packets. Analytically, we proved that, our scheme is very efficient in terms of throughput, end to end delay and coding gain.

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