3D Compressive Sensing for Nodes Localization in WNs Based On RSS

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Abstract—**Compressive sensing (CS) intends to recover signals at a sampling rate significantly (much) lower than that classically used according to the Nyquist theorem. This allows avoiding unnecessary sampling and complexity. In this paper, a Three-Dimensional Compressive Sensing (3D-CS) approach is proposed for nodes localization in wireless networks. In 3D-CS-R2S2 approach, which is based on the ratio of received signal strength (RSS), a 3D sparsity basis and a 3D measurement matrix are used as radio map and noisy measurements respectively in order to recover the target position. A specific multi-linear algebra procedure was developed using N-way array products, together with an adequate decomposition. Both allow formulating the localization problem in a way that is solvable by an ℓ_1-minimization algorithm based on CS theory. 3D-CS-R2S2 improves localization accuracy even if propagation conditions change significantly and/or the effective isotropic radiated power (EIRP) is unknown. Additionally, it enables practical Real Time Localization Systems (RTLS) development since 3D-CS-R2S2 can be functional with a reduced number of base stations without compromising position recovery accuracy. The simulation results show the efficiency of the method that not only succeeds to recover a target position but also improves localization accuracy in presence of noise.**

Keywords-component; Compressive Sensing; Wireless Network, Real Time Localization System, Received Signal Strength

I. INTRODUCTION

Positioning capacities are provided by systems commonly called real time localization systems (RTLS) which can be based on several technologies including Wireless Network (WN) infrastructures. Among signal measures frequently used in WN-based RTLS [1], the received signal strength (RSS) metric can be the best compromise in terms of cost and precision. Several empirical and theoretical models [2] have been proposed to correlate the path loss with the distance between a transmitter and a receiver. Still, this correlation is dependent on the propagation model used for a specific environment and so distance calculation errors will arise if the environment outline or its propagation conditions change. RSS-based localization techniques are therefore primary appropriate where no severe effects influencing propagation, such as multipath, fading or shadowing, occur.

Recently, we proposed a new method [3, 4] based on the ratio of RSS which allows minimizing localization errors due to medium propagation change. The method, called TR2S2 [4] takes advantage of the fact that the ratio of RSS from a pair of Base Stations (BS) varies little when the environment changes as the RSS from either BS increases or decreases simultaneously. TR2S2 uses the ratio $\mathrm{RSS}_1/\mathrm{RSS}_2$ from a BS_1 and a BS_2 as an intrinsic data entity to pinpoint a Reference Point (RP) in the radio map of the environment.

In this work we extend the TR2S2 method by using Compressive sensing (CS) [5, 6]. To the best of our knowledge, 3D-CS-R2S2 is the first method to use three-dimensional compressive sensing theory for node localization in wireless networks. A three-dimensional radio map is created during the training phase of 3D-CS-R2S2, by recording at every RP, the ratio of RSS from each pair of BSs. The RTLS intervenes afterwards in the online phase to estimate, with a high accuracy and speed, a target position based on a few measurements from the BSs that are present in the radio range. This is possible by using CS. In fact, CS makes it possible for 3D-CS-R2S2 to recover target positions as a sparse vector from a small number of noisy measurements in a polynomial time. 3D-CS-R2S2 short execution time also allows ruling out the effects of target mobility which can gravely undermine the localization accuracy. Unlike RSS-based proximity methods such as Fingerprint [7], 3D-CS-R2S2 allows determining a node location with comparable accuracy when the environment changes. Moreover, the prior knowledge of the Effective Isotropic Radiated Power (EIRP) in not required for 3D-CS-R2S2 when homogeneous WNs are used, cancelling in this way another source of errors.

Our contributions in this work are as follows: a) we investigate the use of a 3D radio map, which is less sensitive to environment changes, by using a three-way array data structure; b) we formulate mathematically the problem of nodes position estimation in WNs based on a 3D radio map, as a sparse recovery problem that is solvable by CS theory; c) we develop an appropriate algebraic procedure using tensor products, and also a decomposition which allows node position recovery thanks to ℓ_1-minimization algorithms based on CS theory; d) finally we performed thorough simulations to show that 3D-CS-R2S2 allows target position recovery in polynomial time, even if the propagation conditions of the environment change.

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The rest of the paper is organized as follows. The next section provides some background on CS and selected related works of CS application on both wireless WN-based RTLS and for 3D arrays reconstruction. In Section 3 we formulate the localization problem using a 3D radio map and describe how it is solved using CS. We show simulation results of the recovery performance in Section 4. Section 5 concludes the paper.

II. RELATED WORKS ON COMPRESSIVE SENSING

A. Compressive Sensing backgroud

CS is a recent framework allowing signal acquisition and reconstruction with significantly fewer samples than required by the Shannon/Nyquist sampling theorem. CS is enticing for ill-posed problems since it enables, in polynomial processing time, reconstructing original data via ℓ_1-minimizations with an excellent accuracy even with noisy measurements.

Let's consider a signal represented by $\hat{x}$ an $N \times 1$ column vector and a $N \times N$ basis matrix $\Psi = [\Psi_1, \Psi_2, \dots, \Psi_N]$, generally orthonormal, where Ψ_i are $N \times 1$ column vectors. The signal has a K-sparse representation in basis Ψ if $\hat{x}$ can be approximated by $\hat{x} \approx \sum_{i=1}^{K} \theta_i \times \Psi_i$ with θ_i being the coefficients of $\hat{x}$ in basis Ψ, and $K \ll N$. In order to lift the waste in terms of sampling and complexity, compressed sensing modifies the framework for data acquisition. CS considers an $M \times N$ measurement matrix Φ, incoherent with Ψ, where $M \ll N$. The $M \times 1$ column vector measurements $\hat{y}$ are linear projections of $\hat{x}$ onto Φ, $\hat{y} = \Phi \times \hat{x}$, by substituting $\hat{x}$ by its value, then: $\hat{y} = \Phi \times \Psi \times \theta = \Theta \times \theta$, where Θ is the $M \times N$ matrix. This way, only M measures are expected to recover $\hat{x}$ almost surely. Since $M \ll N$, recovery of $\hat{x}$ from the compressive samples $\hat{y}$ is underdetermined. For a K-sparse signal and under certain conditions on Φ and Ψ (incoherence), one shows that exact signal recovery is possible, with a high probability, by using existing reliable recovery methods based on the ℓ_1-minimization problem formulated as follow ;

$$\theta^* = arg\min\|\theta\|_1 \ subject\ to\ \Phi \times \Psi \times \widehat{\theta^*} = y \quad (1)$$

Finally, the recovered signal is expressed as $\hat{x} = \Psi \times \theta^*$. In other words, in compressed sensing theory, a K-sparse vector of length N can be recovered from only M random measurements, $M \geq \mathcal{C} \times K \times log(N/K)$ where $\mathcal{C}$ is some constant depending on each instance [8].

B. CS in Wireless Networks

The use of CS in Wireless Sensors Networks (WSNs) has stimulated an increasing attention. The advent of CS theory has led to the emergence of new solutions for the problems of efficiently sensing, gathering transmitting, and sharing information from or among a great number of distributed nodes [9]. Recently in WSN research area, many new CS-based methods have been developed for: data routing schemes [10], networks monitoring, sparse event detection, distributed sensing and compression, etc. An early contribution is [11], where a mobile cooperative network is tasked with collecting information from its environment. This paper considers scenarios where mobile intelligent network is in charge of building a map of the spatial variations of one or more parameter(s) cooperatively. More recently, Feng & all [12] have proposed an RSS-based localization method using CS theory. Authors formulated the node localization as a sparse matrix recovery problem in the discrete spatial domain, and used CS theory via spatial sparsity. After a pre-processing procedure of matrix orthogonalization to induce incoherence, target location recovery is performed by solving ℓ_1-minimization programs such as Basis Pursuit (BP) [13], Basis Pursuit Denoising, and Dantzig Selector (DS) [14]. Then a post-processing procedure was used to compensate for the spatial discretization caused by a grid assumption. Although the work in [12] uses CS for target detection, the approach still uses RSS measures directly and only differs from the Fingerprint method by using a CS-based location recovery The objective of our work is to overcome the deficiency of RSS based radio mapping by investigating the tradeoffs between the generation of a generic radio map which is less sensitive to environment change, while still using the RSS metric. Furthermore, the work analyzes the 3D-CS based RTLS performance in order to improve nodes localization accuracy.

C. Three-dimensional CS.

Three-Dimensional CS (3D-CS) has recently emerged for 3D shape and image reconstruction in 3D space. The majority of applications that investigate the 3D-CS approach lie in the image processing research area, especially for Magnetic Resonance imaging (MRI), Radar, and Ultrasonic imaging applications. In [15], authors apply CS to obtain 3D scene reconstruction using ultrasonic sensing capabilities. By transmitting incoherent wideband ultrasonic pulses and receiving their reflections, a sensor array can sense the scene and reconstruct it using standard CS reconstruction algorithms. In MRI true 3D-CS reconstruction are reputed to take a very long time [16]. Others authors [17] have suggested separating the 3D problem into many 2D problems to reduce computation time and memory load, in a similar manner to interferometry principle. While this approach can improve computational performance, powerful 3D connectivity information may inherently be lost.

In this paper a 3D-CS method is proposed for target position recovery based on a 3D radio map. A full 3D approach is investigated, by using n-way arrays in multi-linear algebra, to confirm its adequacy in terms of execution time and position recovery performance.

III. A 3D-CS BASED TARGET LOCALIZATION METHOD

A. Problem formulation

In a previous work, a localization technique called TR2S2 [4]. TR2S2, has a number of strong points, inter alia, being cost effective as it is based on the RSS which can be measured by low-cost receivers, in addition to its localization capability with unknown EIRP. As a matter of fact, when environments change, even if the Transmitter Receiver (T-R) distance is unchanged, the RSS still may change significantly. TR2S2 avails against environment non homogeneity at an instant t. This is because TR2S2 makes use of RSS ratios, and the ratio of RSS from a pair of BSs varies little with environment change as the RSS from either base stations increases or decreases simultaneously. Simulations demonstrated TR2S2 performance when using various radio propagation models, for some of the main wireless technologies currently available

(802.11x, 802.16x, etc.), and within different localization areas (urban, suburban, etc.) ranges.

In classical localization methods, target nodes location has usually been computed with geometric calculation, as is the case with TR2S2, or by using methods such as K-Means, K-Nearest Neighbors and centroid techniques, etc. The originality of the proposed 3D-CS-R2S2 method resides in applying CS theory for target node position recovery, as well as handling n-way arrays to address the three-dimensionality for each measurement. In other words, each input data connects three wireless nodes, namely, two reference points and one possible target. The next section provides a formal expression of the proposed 3D-CS-R2S2 recovery technique.

B. 3D-CS recovery program

Most RSS-based RTLS need a measurement-based training phase, commonly called Fingerprinting, during which the radio map of the environment is created. Conventional radio maps compute and store the average of the maximum number of RSS measurements samples over time, from all existing BSs at many RPs, with known coordinates, covering optimally the entire detection area. This process is costly and may be insufficient especially for outdoor environments, since it will be practically impossible to identify all the propagation conditions.

- Sparsity Tensor $\Psi_{K\times K\times N}$

In 3D-CS-R2S2 , each RSS ratio is computed according to RSS measurements at $\mathrm{RP_n}$ from two base stations $\mathrm{BS_i}$ and $\mathrm{BS_j}$. Then radio map database can be represented with Ψ:

$$\Psi_{K\times K\times N} = \begin{pmatrix} \psi_{1,1,n} & \cdots & \psi_{1,M,n} \\ \vdots & \ddots & \vdots \\ \psi_{M,1,n} & \cdots & \psi_{M,M,n} \end{pmatrix} \quad (2)$$

And $\psi_{i,j,n} = \widehat{\psi_n} = [\psi_{i,j,1},\ \psi_{i,j,2}, \dots, \psi_{i,j,n}]$ (3)

Where $\psi_{i,j,n}$ is the ratio of recorded RSS measurements from BS_i divided by that from BS_j recorded at RP_n, for $i = 1,2,\dots,K, j = 1,2,\dots,K$, and $n = 1,2,\dots,N$. K is the total number of BSs and N is the total number of RPs taken in the detection area. Thereby, the radio map is a 3-way array known as tensor in mode-1 of $(x_k, y_k; \widehat{\psi_n})$, for$n = 1,2,\dots,N$, where (x_n, y_n) are Cartesians coordinates of the n^{th} RP. Tensor methods have been introduced since the 1960's for multi-dimensional data analyses.

- Target position vector $\theta_{N\times 1}$

Let all RPs in the detection area be represented by a column vector θ of size K. At a given time, each target node position has a unique position in the discrete spatial domain; here represented as a T-sparse vector with all cells are equals to zero expect $\theta_n = 1$, where n is the index of the RP at which the target node is located, and T is the total number of target nodes;

$$\theta = [\theta_1, \dots, \theta_{n-1}, 1, \theta_{n+1}, \dots, \theta_N]^T \quad (4)$$

The localization problem is stated as a sparse signal problem for which CS can be applied for target node position recovery, which is equivalent to reconstructing vector θ^* from a set of noisy measurements.

- Measurement matrix $\Phi_{M\times N}$

During on-line localization, a target node will catch sight of only a small number K of BSs randomly situated in the detection area, $M < K \ll N$. The matrix $\Phi_{M\times N}$ represents the positions of BSs belonging to the target read range. $\Phi_{i\times j} = 1$ means that $\mathrm{BS_i}$ is located at $\mathrm{RP_j}$. Since Φ is created randomly, then different online tensor measurements $\mathrm{yTesnor_{M\times M\times T}}$ are created at different runs. $yTesnor_{i\times j\times k}$ is the current recorded ratio RSS_i/RSS_j measured form BS_i and BS_j respectively at target node T_k level.

Let us recapitulate with the following model:

$$yTensor = \Psi \times_{[1,2]} \Phi \times \theta + \varepsilon \quad (5)$$

Where $\theta \in \mathbb{R}^n$is the vector of target position to recover, $yTensor \in \mathbb{R}^{M\times M\times N}$ is the observations tensor (current/online measurements), $\Psi \times_{[1,2]} \Phi \in \mathbb{R}^{M\times M\times N}$is a measurement tensor with $M \ll N$, and ε is the measurement noise. In order to recover θ via ℓ_1-minimization algorithm, CS theory imposes two essential conditions, namely the sparsity of θ and the incoherence of Ψ and Φ [6]. The first condition is already proved in (3). Because Ψ and Φ are artlessly coherent in spatial domain, each element of the Φ can be represented linearly by combining a few number of columns of the basis Ψ. To comply with the second condition, an orthogonalization procedure is required to make Ψ and Φ incoherent so that CS works properly. Since the concept of incoherence is not defined for orthogonal matrices, in the following section, we present an efficient computational transformation allowing the orthogonalization of n-way arrays so that Φ will a have dense representation in the basis Ψ.

First, let assume $y = T \times yTensor$, and let;

$$T = Z \times_n A^\dagger \quad (6)$$

Where $A = \Psi \times_{[1,2]} \Phi$, i.e. the tensor Ψ is successively multiplied in mode-1 and mode-2 by matrix Φ, and Z is the transpose of High Order Singular Value Decomposition (HOSVD) of A^T [18, 19]. HOSVD models allow to decompose a three dimensional tensor $\mathcal{T}$ into one core entity $\mathcal{G}$ and three matrices factors (7), as illustrated in Fig.1;

$$\mathcal{T} = [\![\mathcal{G}; A, B, C]\!] \approx \mathcal{G} \times_1 A \times_2 B \times_3 C \quad (7)$$

The most attractive features of HOSVD are: (a) matrix factors $(A; B; C)$ are generally orthogonal, (b) core tensor $\mathcal{G}$ must have mutually orthogonal slices in all the three modes of $\mathcal{T}$. The operator $\times_n$ in (6) signifies the product of tensor Z's core and the mode products of the three matrix factors of tensor $A^\dagger$, in the same way as in (7).

$$\begin{aligned} y &= Z \times_n A^\dagger \times yTensor \\ &= Z \times_n A^\dagger \times A \times \theta + Z \times_n A^\dagger \times \varepsilon \\ &= Z \times \theta + Z \times_n A^\dagger \times \varepsilon = Z \times \theta + \varepsilon' \end{aligned} \quad (8)$$

Owing to the transformation formulated in (8), tensor Z is orthogonal, and the incoherence condition is now fulfilled. Accordingly, given a set of measurements y, we can recover θ (or its approximation θ^*) by solving an ℓ_1-minimization problem in accordance with CS approach, in polynomial time.

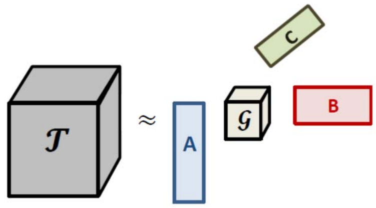

Figure 1. Tucker decomposition.

The problem posed in (8) can be solved efficiently using several methods proposed in recent years. Our focus in this paper will be only on the well-known Basis Pursuit (BP) method, just to validate and prove our 3D CS approach. In fact, BP can be used with noisy data by solving an optimization problem trading off a quadratic misfit measure with an l_1 norm of coefficients occurring in the representation that we seek to recover. In our case, BP method refers to the solution of:

$$\theta^* = \arg\min_{\theta} \|\theta\|_1 \ \textit{subject to}\ Z \times \theta = y \tag{9}$$

The vector θ^* returned when solving (9) by BS is assumed to be a 1-sparse $N \times 1$ vector with all cells equal to zero expect $\theta^*_n = 1$, where n is the index of RP at which the target node is located. Every once in a while, it may happen that the target node is not situated exactly at a RP coordinate, and otherwise, the recovered θ^* is as a matter of fact an approximation of the real data, leading to have the θ^* vector containing a few non-zero coefficients. In that event, the recovered target position $Q^{\otimes}$ is the centroid of the positions of index with highest coefficients satisfying this constraint: (10)

$$Q^{\otimes} = Centroid\big((x_h, y_h) | \theta^*(h) \geq \max(\theta^*(h) \times 0.95)\big)$$

IV. SIMULATIONS RESULTS

For the purpose of simulations, the free space shadowing model [2] is used to predict the received signal strength when the transmitter and receiver are in Line Of Sight (LOS). The received power (P_r) is given by the Friis equation as follow:

$$P_r = \frac{P_t\, G_t\, G_r\, \lambda^2}{(4\pi)^2\, d^n\, L} \tag{11}$$

Where P_t is the transmitted power, G_t and G_r are the gains of the transmitter and receiver antennas and λ is the wavelength. L is the system loss, not related to propagation. Here, G_t, G_r and L are set to 1 (matched antennas and lossless system). Path loss exponent n is the key parameter in the localization algorithm that defines the rate at which RSS decreases with distance in a specific environment. Let $P_{r_i} = \frac{P_t\, G_t\, G_r\, \lambda^2}{(4\pi)^2\, {d_i}^n\, L}$ and $P_{r_j} = \frac{P_t\, G_t\, G_r\, \lambda^2}{(4\pi)^2\, {d_j}^n\, L}$ be the RSS values from two base stations BS_i and BS_j at distances d_i and d_j respectively from the target node when it is located at RP_n.

$$\Psi_{n \times i \times j} = \frac{Pr_i}{Pr_j} \tag{12}$$

In the presented simulations, we consider 8 BSs and 4 target nodes randomly situated in a detection area of 12m × 12m size. BSs can be situated outside the detection area as long as they remain in the targets read range. The measurement matrix is returned according to BSs detected in the read range. The simulations aim, on the one hand to validate the results of the 3D-CS-R2S2, and on the other hand to analyze the effects of the two major sources of error the localization accuracy; propagation conditions change and target mobility.

In the first scenario, target nodes are assumed to move discontinuously across the RPs positions with no propagation conditions of environment changing. In this ideal case, the 3D-CS-R2S2 localization program manages to recover multiple targets positions accurately, even in special cases such as a target located exactly at BS coordinates, as illustrated in Fig.2.

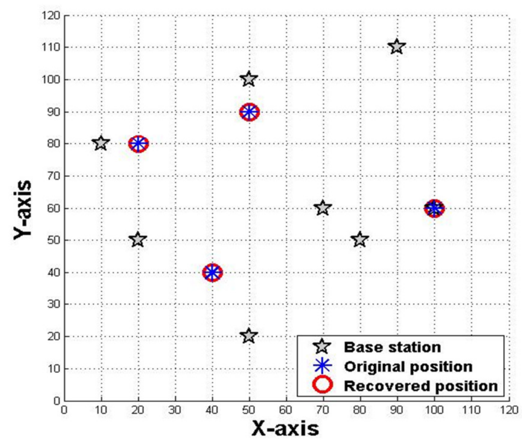

Figure 2. 3D-CS localization of targets at discrete positions and no propagation conditions changing

In the second scenario, target nodes move over the grid randomly, and propagation conditions remain unchanged. The values in the measurement matrix are no longer exactly equal to those in radio map, and the recovered position is mostly the centroid of several potential positions. In this case, the 3D-CS-R2S2 method demonstrates as well good performance, recovering targets positions substantially close to the originals, as shown in Fig.3.

In the third scenario, measurement noises are introduced by adding a random shadowing, resulting in additional fluctuations of the RSS measurements. The additional shadowing is expressed by several standard deviations $\sigma = 1.5,3,4,5,6$ in dBm. Fig.4 demonstrates that localization accuracy does not degrade significantly when the propagation conditions change, hence the advantage of using the ratio of RSS (R2S2) instead of simple RSS. The positioning mean error curve in Fig.4 varies lightly with a slight slope to the rise when the shadowing effect increases. The localization mean error increases from 1.11m to 1.535m when the standard deviation σ increases from 1.5dBm to 6dBm.

Execution time is one of the most important criteria when implementing RTLS. Tab.1 summarizes the execution time of the 3D-CS-R2S2 for the localization of 1 target node and for 4 targets nodes simultaneously in the 12m × 12m grid. As can be seen, execution time for the localization of one target is slightly lower than for 4 targets. Looking into detail, the execution time to perform HOSVD task is approximately the same because the number of BSs is maintained at 8, and thus the size of measurement matrix $\Phi_{M\times N}$ is the same. Contrary to HOSVD time, the execution time of BP algorithm is significantly lower in the first case. Indeed, the algorithm is run as many times as the number of targets. As a concluding remark on this aspect, the total execution time is proportional to the number of BSs and target nodes involved in the localization.

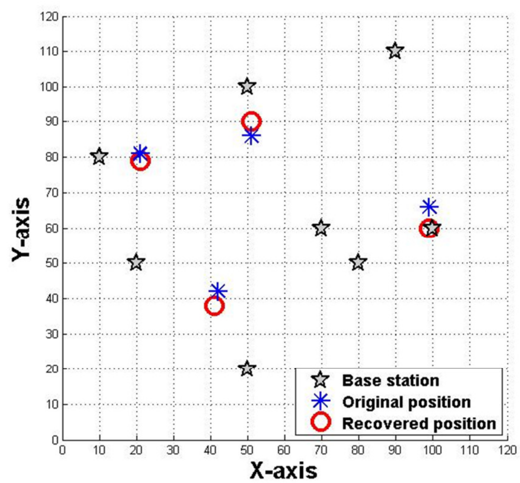

Figure 3. 3D-CS localization of targets at continuous positions and no propagation conditions changing

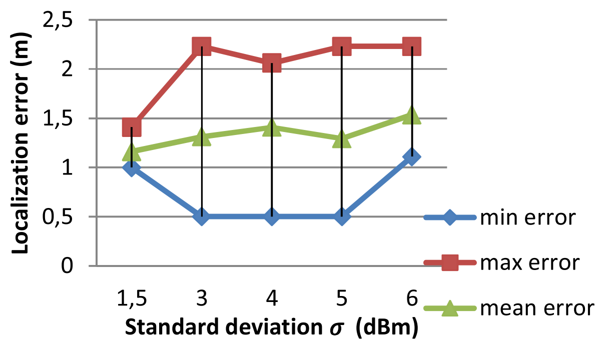

Figure 4. 3D-CS localization error as function of standard deviation σ.

Table 1. Execution time details

Number of targets	Ψ load time(s)	HOSVD time(s)	BP time(s)	Total time(s)
1	0.0624	59.8	0.889	60.7514
4	0.078	62.022	1.638	63.738

V. CONCLUSION

In this paper we focused on a full three-dimensional CS approach for mobile nodes localization in wireless networks. Unlike conventional RSS based localization methods, the proposed method is deterministic and fully founded on multi-linear algebra, instead of geometric computation to mark off the estimated boundaries of the targets. Since the ratio of received signal strengths from pairs of base stations is selected as a data to characterize a node presence at a reference point, the radio map and the observation matrix used to set up input data in 3D-CS-R2S2 are represented by three-way arrays called tensors. An appropriate algebraic procedure is developed using tensor products together with a suitable decomposition which allow solving the wireless nodes localization problem by ℓ_1-minimization algorithms based on CS theory. Simulation results show that almost exact target position recovery is possible, with a high probability, and even if the propagation conditions of environment change and/or the target node is mobile.

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