

The Bright Side of Coulomb Blockade: Supplementary Material

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DETAILS ON SETUP AND CALIBRATION

We describe the calibration of the low frequency circuitry for voltage bias and current measurement and the microwave components used to define the environment of the junction and to measure the emitted radiation.

Low frequency circuit

In addition to the components depicted in Fig. 1 in the paper, the low frequency circuit includes multipole RC low pass filters in the bias line (cutoff at 8 kHz) and the current measurement lines (cutoff at $\simeq 100$ kHz), several capacitors to ground and a copper powder filter between readout resistor and bias T (see Fig. 5).

Calibration

Calibration of the division factor of the voltage divider used for biasing the sample is performed in situ by reading out the applied bias voltage through one of the lines used for current measurements (see Fig. 1). We use a lock-in technique for this measurement and minimize the critical current of the junction so that the current through the sample is negligible.

The 1 k Ω NiCr resistor used for current measurement cannot be calibrated in situ due to large lead resistances, but we have verified that a resistor from the same batch does not deviate from its nominal value by more than 1% at the operating temperature of our refrigerator.

In the presented measurements we have corrected the applied bias voltage for the voltage drop over the 1 k Ω resistor.

The good agreement between theory and data in Fig. 4 supports the accuracy of this DC calibration.

Microwave circuit

The microwave chain (see Fig. 5) consists of the junction, the resonator, a bias T, a 6 dB attenuator, a triplexer with crossover frequencies 4 and 8 GHz, two 4 to 8 GHz isolators at base temperature, an isolator at 700 mK, a cryogenic amplifier at 4 K with 3.5 K noise temperature, a 3 dB attenuator at 4 K followed by a room temperature amplification and measurement chain. The

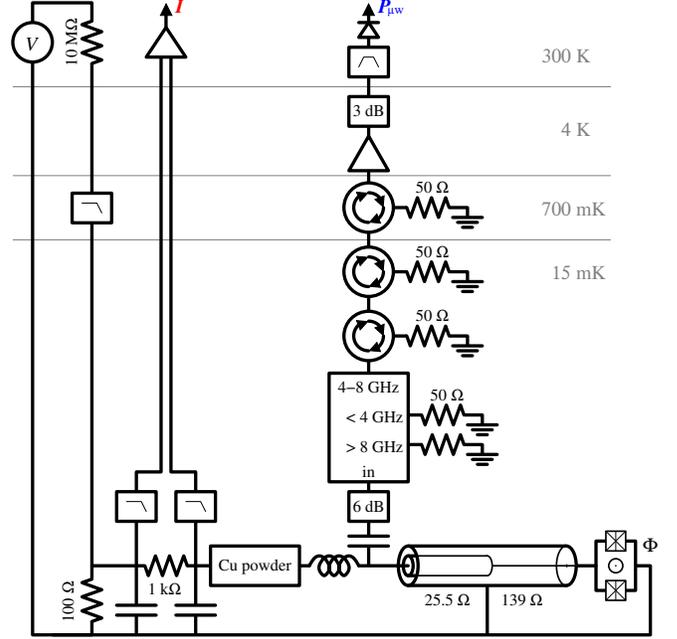


Figure 5: More detailed view of the experimental setup. Only circuit components inside the refrigerator are shown in full detail.

isolators protect the sample from the amplifier noise in their working range from 4 to 8 GHz. The triplexer, working from DC to 18 GHz, connects the sample to the microwave chain only in the 4 to 8 GHz range. Outside this range the sample is connected to cold 50 Ω loads so that the sample “sees” a cold 50 Ω load throughout the 18 GHz frequency range of the triplexer. Reflections due to the insertion loss of the triplexer are suppressed by the 6 dB attenuator.

Calibration

The resonator impedance $Z(\nu)$ and the gain of the microwave chain from the sample to the top of the refrigerator have to be calibrated in situ because the superconducting resonator must be cooled below its critical temperature and the amplifier gain and cable attenuation depend on temperature. We perform both calibrations using quasiparticle shot noise as white current noise reference. We bias the junction at 2 mV, far above the gap voltage 0.4 mV of our junction, where the current noise is in good approximation $S_{II} = eI$ at frequencies

$|\nu| \ll eV/h \simeq 0.5$ THz. In order to separate this noise from the noise floor of the cryogenic amplifier, we then apply small variations of the bias voltage and measure the corresponding changes in the measured microwave power with a lock-in amplifier.

The conversion of S_{II} into emitted microwave power depends on the environment impedance $Z(\nu)$ seen by the junction and its tunneling resistance R_N . First, only a fraction $R_N^2/|R_N + Z(\nu)|^2$ of the current noise is absorbed by the environment, but this factor is close to 1 (> 0.85) as $R_N = 17.9$ k Ω is much larger than the environment impedance. The current noise in the environment has then to be multiplied by $2\text{Re} Z(\nu)$ to obtain the microwave power emitted by quasiparticle shot noise:

$$S_P(\nu) = 2eV \frac{\text{Re} Z(\nu) R_N}{|Z(\nu) + R_N|^2}. \quad (6)$$

The emitted power depends on $Z(\nu)$, therefore $Z(\nu)$ and the gain of the microwave chain cannot be calibrated independently. We solve this problem by making the following assumptions:

1. The gain of the microwave chain is frequency independent in the 5 to 7 GHz range.
2. The integral over the peak in $\text{Re} Z(\nu)$ from 5 to 7 GHz corresponds to calculations based on the resonator geometry.

Assumption 1 is justified by a previous calibration of the gain of the amplifiers, found to be constant in the 5 to 7 GHz range within 0.5 dB, and by observing that the noise floor of the amplifier in this experiment in the same 5 to 7 GHz range deviates by less than 0.5 dB from its mean value, meaning that the gain and attenuation after the input of the cold amplifier are almost frequency independent. However, frequency dependent attenuation between the sample and the cryogenic amplifier is not detected this way and can be source of slight inconsistencies.

Assumption 2 relates to the fact that the integral over $\text{Re} Z(\nu)$, proportional to the characteristic impedance of the resonance, mainly depends on the transmission line impedance of the quarterwave segment closest to the junction, less on the second segment, and only very weakly on the load impedance beyond the resonator. For example, the characteristic impedance of the resonance changes by less than 20% when the the load impedance is changed from 50 Ω to any value between 20 Ω and infinity. Therefore we can get an accurate estimate of this integral from the impedance of the resonator segments, even in the presence of imperfections in the line impedance seen from the resonator. We calculate the characteristic impedance of our quarterwave segments from the designed resonator geometry, checked with an electron microscope and the well known stackup of our sample. We obtain characteristic impedances 139 Ω and 25.5 Ω ,

resulting in a characteristic impedance of the resonance mode of 150 Ω . The fact that $P(E)$ theory using this resonator impedance accurately fits the relative heights of first and second order peaks in Fig. 2 indicates that this estimation of the resonator impedance is correct.

With these two assumptions we can use Eq. (6) to separately calibrate the power gain and $Z(\nu)$. To calibrate the power gain in Figs. 2 and 3 we compare the measured shot noise signal to Eq. (6) integrated over the bandwidth of our measurement ranging from 5 to 7 GHz.

In order to determine the spectral dependence of $Z(\nu)$ we use a narrow band pass filter (as for the data in Fig. 4) to measure a signal proportional to S_P . To extract $\text{Re} Z(\nu)$ from the emitted power using Eq. (6), one would need to know $\text{Im} Z(\nu)$, which is not measured independently. To circumvent this difficulty, we use Eq. (6) iteratively to calculate $Z(\nu)$: we start with the calculated impedance $\text{Re} Z_0(\nu)$ in the denominator and in each step calculate a new $\text{Re} Z_n(\nu)$, normalize it using assumption 2, and then calculate $\text{Im} Z_n(\nu)$ using the Kramers-Kronig relations with $\text{Re} Z_n(\nu)$ in the 5 to 7 GHz range where S_P was measured and $\text{Re} Z_0(\nu)$ outside this range. This procedure converges quickly and yields a precise determination of $\text{Re} Z(\nu)$ because $\text{Im} Z(\nu) \ll R_N$ and therefore only has a small influence on the extracted $\text{Re} Z(\nu)$.

NOISE EMITTED BY A JOSEPHSON JUNCTION IN THE DCB REGIME

We give here two derivations of expression (5) in the paper. The first one follows the derivation of Coulomb blockade in [4] and treats photon emission as fluctuations in the Cooper pair tunneling rate. The second derivation explicitly evaluates photon emission and absorption rates in a mode of the electromagnetic environment.

Derivation 1

We derive the emission noise density $S_{II}(\nu)$ of the current through the voltage biased Josephson element, which yields the microwave power emission density $S_P(\nu) = 2\text{Re} Z(\nu)S_{II}(\nu)$.

The Josephson element is described by the Hamiltonian $\hat{H}_J = -E_J \cos \hat{\delta}$ and the current operator through the element is

$$\hat{I} = -\frac{2e}{\hbar} \frac{\partial \hat{H}_J}{\partial \delta} = i \frac{e}{\hbar} E_J \left(e^{i\hat{\delta}} - e^{-i\hat{\delta}} \right)$$

The current-current correlator function is then

$$\begin{aligned} S_{II}(t) &= \langle \hat{I}(t) \hat{I}(0) \rangle \\ &= \frac{e^2 E_J^2}{\hbar^2} \left(\langle e^{i\hat{\delta}(t)} e^{-i\hat{\delta}(0)} \rangle + \langle e^{-i\hat{\delta}(t)} e^{i\hat{\delta}(0)} \rangle \right). \end{aligned} \quad (7)$$

To arrive at the last line we have noted that at non-zero dc bias voltage V terms $e^{\pm i\hat{\delta}(t)}e^{\pm i\hat{\delta}(0)}$ with the same sign in the exponential average to zero. We decompose the phase difference across the tunnel element $\hat{\delta}(t) = \delta_0(t) + \tilde{\delta}(t)$ into the deterministic part $\delta_0(t) = 2eVt/\hbar$ and a fluctuating random phase $\tilde{\delta}(t)$ caused by the fluctuations in the electromagnetic environment. We use the gaussian character of the noise of the linear environment to rewrite [4]:

$$\begin{aligned} \left\langle e^{\pm i\hat{\delta}(t)} e^{\mp i\hat{\delta}(0)} \right\rangle &= e^{\pm i2eVt/\hbar} \left\langle e^{\pm i\tilde{\delta}(t)} e^{\mp i\tilde{\delta}(0)} \right\rangle \\ &= e^{\pm i2eVt/\hbar} e^{J(t)} \end{aligned}$$

where $J(t) = \left\langle \left(\tilde{\delta}(t) - \tilde{\delta}(0) \right) \tilde{\delta}(0) \right\rangle$ is the (superconducting) phase-phase correlation function across the impedance, so that (7) becomes

$$S_{II}(t) = \frac{e^2 E_J^2}{\hbar^2} 2 \cos \frac{2eVt}{\hbar} e^{J(t)}.$$

The (non-symmetrized) current noise density $S_{II}(\nu)$ is directly the Fourier transform of the current-current correlator $S_{II}(t)$

$$S_{II}(\nu) = \int_{-\infty}^{\infty} S_{II}(t) e^{-i2\pi\nu t} dt$$

(Wiener Khinchin theorem) which yields immediately

$$S_{II}(\nu, V) = \frac{2\pi e^2 E_J^2}{\hbar} (P(2eV - h\nu) + P(-2eV - h\nu))$$

with $P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} e^{J(t)+iEt/\hbar} dt$ the Fourier transform of $\exp J(t)$. Assuming that the mode of the environment at frequency ν is in the ground state, i.e. $kT \ll h\nu$, the current noise gives rise to a photon emission rate density

$$\begin{aligned} \gamma &= \frac{S_P(\nu)}{h\nu} = \frac{2 \operatorname{Re} Z(\nu)}{h\nu} S_{II} \\ &= \frac{2r(\nu)}{\nu} \frac{\pi}{2\hbar} E_J^2 (P(2eV - h\nu) + P(-2eV - h\nu)). \end{aligned}$$

In expression (5) in the paper we have neglected the second term corresponding to photon emission during backward tunneling which is negligible for $kT \ll 2eV + h\nu$ as in our experiment.

Derivation 2

The Hamiltonian of the total system is

$$\hat{H} = \hat{H}_{\text{em}} + \hat{H}_J$$

with

$$\begin{aligned} \hat{H}_{\text{em}} &= \sum_l \frac{\left(\frac{\hbar}{2e} \hat{\phi}_l \right)^2}{2L_l} + \frac{\hat{Q}_l^2}{2C_l} \\ \hat{H}_J &= -E_J \cos \hat{\delta} \\ \hat{\delta} &= \frac{2eVt}{\hbar} + \sum_l \hat{\phi}_l. \end{aligned}$$

Here \hat{H}_{em} is the Hamiltonian describing the environment of impedance $Z(\omega)$. C_l and L_l describe the mode at frequency $\nu_l = 1/2\pi\sqrt{C_l L_l}$. A second constraint on C_l and L_l imposed by the requirement that the sum of the resonator impedances approaches $Z(\nu)$, giving

$$Z_l = \sqrt{\frac{L_l}{C_l}} = \frac{2}{\pi} Z(\nu) \frac{\Delta\nu}{\nu},$$

where $\Delta\nu$ is the spacing between adjacent modes, supposed to be constant. The Cooper pair tunneling rate can now be calculated using Fermi's golden rule:

$$\begin{aligned} \Gamma^{\rightarrow} &= \frac{\pi}{2\hbar} E_J^2 \sum_{\substack{n_l \geq 0 \\ n_l + m_l \geq 0}} \prod_l p_{l, n_l} \left| \left\langle n_l + m_l \left| e^{i\hat{\phi}_l} \right| n_l \right\rangle \right|^2 \\ &\quad \delta \left(\sum_l m_l h\nu_l - 2eV \right) \\ &=: \frac{\pi}{2\hbar} E_J^2 P(2eV), \end{aligned}$$

where $p_{l, n}$ is the probability for mode l to be in state $|n\rangle$ before the tunneling event. We assume that the tunneling rate is low so that the environment can reach thermal equilibrium before each tunneling event and the $p_{l, n}$ describe thermal equilibrium states. This equation can be seen as the definition of $P(E)$, the probability density for the environment to absorb energy E when a Cooper pair tunnels. Calculating $P(E)$ by evaluating this sum is possible but cumbersome, so usually the approach from Ref. [4] is taken.

In order to calculate the rate for emitting photons at frequency ν we treat mode k at $\nu_k = \nu$ separately and keep track of the photon number difference m_k . We absorb all other modes into a function $P'(E)$ describing a modified environment where mode k has been removed.

$$\Gamma_m^{\rightarrow} = \sum_{n=\max\{0, -m\}}^{\infty} p_{k, n} \left| \left\langle n+m \left| e^{i\hat{\phi}_k} \right| n \right\rangle \right|^2 \frac{\pi}{2\hbar} E_J^2 P'(2eV - mh\nu_k).$$

In this expression the phase operator $\hat{\phi}_k$ across the resonator k can be expressed in terms of the rising and lowering operators a_k^\dagger and a_k :

$$\hat{\phi}_k = \sqrt{\rho} (\hat{a}_k^\dagger + \hat{a}_k)$$

with

$$\rho = \pi \frac{4e^2}{\hbar} Z_k = 2 \frac{4e^2}{\hbar} \operatorname{Re} Z(\nu) \frac{\Delta\nu}{\nu} = 2r(\nu) \frac{\Delta\nu}{\nu}$$

The spectral density of photon emission/absorption is calculated in the limit $\Delta\nu \rightarrow 0$ where $e^{i\hat{\phi}_k} \rightarrow 1 + i\sqrt{\rho}(\hat{a}_k^\dagger + \hat{a}_k)$ and $P'(E) \rightarrow P(E)$. In this limit Γ_m^\rightarrow is nonzero only for $m = 0, \pm 1$. The rate without photon emission or absorption Γ_0^\rightarrow simply tends to the bare Cooper pair tunneling rate, i.e. $\Gamma_0^\rightarrow \rightarrow \Gamma^\rightarrow$. For $m = \pm 1$ we calculate the density of photon absorption/emission

$$\begin{aligned} \gamma_\pm^\rightarrow(\nu) &= \lim_{\Delta\nu \rightarrow 0} \frac{\Gamma_{\pm 1}^\rightarrow}{\Delta\nu} \\ &= 2\frac{r(\nu)}{\nu} \sum_{n=1}^{\infty} n p_{n_{\mp 0}}(\nu) \frac{\pi}{2\hbar} E_J^2 P(2eV \mp h\nu). \end{aligned} \quad (8)$$

The backward tunneling processes Γ^\leftarrow and $\gamma_{\pm 1}^\leftarrow$ have the same expressions as the corresponding forward processes

but with V replaced by $-V$.

At low temperature $kT \ll h\nu$ the mode at ν is in the thermal ground state, i.e. $p_n(\nu) \simeq \delta_{n,0}$, and $P(-2eV - h\nu) \simeq 0$, so that $\gamma_\pm^\rightarrow = \gamma_\pm^\leftarrow = 0$. We then find expression (5) in the paper:

$$\gamma(\nu) = \gamma_+^\rightarrow(\nu) = \frac{2r(\nu)}{\nu} \frac{\pi}{2\hbar} E_J^2 P(2eV - h\nu)$$

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- [4] G.-L. Ingold and Y. V. Nazarov, in *Single charge tunneling*, edited by H. Grabert and M. H. Devoret (Plenum, 1992).